

Intrepid Math

Challenging Common Core Problems

Grades 4 – 6

Set 3

from 5280 Math
by Jerry Burkhart

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Each problem includes four parts.

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***Intrepid* Math**
Challenging Common Core Problems
Quick Start Guide

1. Prepare

Choose one problem.
Make copies.
Gather materials.
Familiarize yourself with the problem.

2. Introduce

Distribute the problem, materials,
and *thinking paper**.
Give directions.
Establish expectations.
Set an estimated time frame.

3. Support

Facilitate discussion or check in
with individuals and groups.
Help students clarify their
thoughts.
Ask guiding questions.
Encourage and motivate students
through the tough spots.
Determine when students have
reached a stopping point.

4. Wrap up

Gather completed work with
explanations.
Acknowledge students' effort
and progress.
Summarize what was learned.
Assess student work.

Advice for students

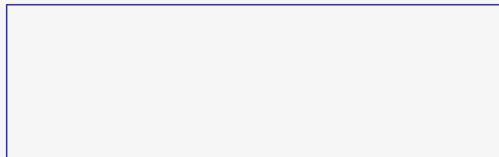
Plan on spending a few days or more on many of the problems.
The problems are challenging. Expect to get stuck and to make some mistakes.
Take your time. It is more important to learn new things than to finish the entire problem.

*See the FAQs to learn more about thinking paper and teaching strategies.

Name _____

Problem 1

Maria has finished a collection of jigsaw puzzles with panoramic pictures. The puzzle pieces are arranged in rows and columns. Each puzzle has the same shape, size, and number of pieces. A scale drawing of the borders looks like this:



Maria has put together 8113 pieces. How many puzzles has she completed, and how many rows and columns are in each puzzle?

Name _____

Problem 1: *Testing the Waters*

Maria has finished a collection of jigsaw puzzles that are almost square. The puzzle pieces are arranged in rows and columns. Each puzzle has the same number of pieces. She has put together a total of 715 pieces. How many puzzles has she completed, and how many rows and columns are in each puzzle?

Name _____

Problem 1: Diving Deeper

Choose one or more questions to answer, or think of your own new questions to explore.

1. Why is there only one realistic solution to Problem 1?
2. Create a similar problem that has only one realistic solution.
3. Choose a number, and create a similar problem that has that number of solutions.
Explain your thinking process.

Problem 1

Teacher's Guide

Topics

Division and multiplication of multi-digit numbers; estimation; factors and multiples; scale drawings

Materials

None

The grade level notes below are a rough guide for working with different age groups. Read through all three grades to get a feel for the likely range of students' thinking. Their approaches may vary a lot, so never feel limited to a particular grade level. Observe your students' ideas and strategies to assess their thinking, and build from there.

Grade 4 Common Core Standards: 4.NBT.B 4.OA.A.4 4.MD.A.3

Ask students to solve this problem without a calculator. If 8113 is too large, begin with Testing the Waters. It's okay if students haven't learned long division procedures yet! They can use repeated subtraction, grouping, decomposing, multiplication, estimation, or other strategies. Encourage them to search for strategies and observations that reduce the amount of dividing that they need to do. See the 'Basic strategies' in the Solutions. Some fourth graders may use ideas from the intermediate or advanced strategies as well.

Grade 5 Common Core Standards: 5.NBT.B.5 5.NBT.B.6

In grade 5, students may use estimation in a more sophisticated way and incorporate strategies related to the ones digits. See the 'Intermediate strategies' in the Solutions, but be aware that students may use other strategies as well.

Grade 6 Common Core Standards: 6.NS.B.2 6.NS.B.4

Some sixth graders may use properties of factors in a deeper way. They may also apply observations about the scale drawing. See the 'Advanced strategies' in the Solutions. The Diving Deeper questions encourage students to think about relationships between factors and prime factors.

Solutions begin on page 37.

Name _____

Problem 2

Flora Flea likes to hop on number lines. She starts at 0.

Part 1: Flora hops forward 1 tenth, then backward 1 tenth as far. She hops a total of six times, going back and forth 1 tenth as far each time. What decimal does she land on? If she keeps doing this, about what fraction of the way to 1 will she be?

Part 2: Flora hops forward only—first 1 half, then 1 half as far each time for a total of six hops. What decimal does she land on? If she keeps doing this, about what fraction of the way to 1 will she be?

Name _____

Problem 2: *Testing the Waters*

Flora Flea likes to hop on number lines. She starts at 0 and hops forward 1 tenth. She hops forward five more times, hopping 1 tenth as far each time. What decimal number does she land on? If she keeps doing this, about what fraction of the way to 1 would you guess that she can get?

Name _____

Problem 2: *Diving Deeper*

Choose one or more questions to answer, or think of your own new questions to explore.

1. Redo Testing the Waters if Flora starts at 1 and jumps backward each time.
2. Redo Part 1 if Flora starts at 1, jumps backward, and reverses direction each time.
3. Redo Part 2 if Flora reverses direction each time.
4. Experiment with other fractions for Flora's jumps. For example, suppose that Flora hops $\frac{1}{4}$ or $\frac{1}{5}$ as far each time. Solve the problems using both decimals and fractions.

Problem 2 Teacher's Guide

Topics

Place value; decimals; visualizing decimals and fractions on a number line

Materials

None

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Grade 4 Common Core Standards: 4.NF.C

Most younger students should begin with Testing the Waters. If they have trouble getting started on either problem, ask them to think about what 1 tenth of 1 tenth is and why. Have them show what 1 tenth and 1 hundredth look like on a number line. As they picture Flora's hopping, suggest that they imagine a magnifying glass that shows more and more detail as they divide the number line into smaller and smaller pieces. Many fourth graders may complete only the Testing the Waters questions or Part 1.

Grade 5 Common Core Standards: 5.NBT.A

Fifth graders may still want to start with Testing the Waters. Encourage them to think about place value and to visualize the distances on the number line instead of using rules for adding and subtracting decimals. See the strategies described in the Solutions. Fifth graders are more likely than younger students to reach Part 2.

Grade 6 Common Core Standards: ---

Sixth graders may put much of their energy into Part 2. Some of them may finish it and work on the Diving Deeper questions as well. Challenge them to think about both decimals and fractions as they jump!

Solutions begin on page 39.

Name _____

Problem 3

The product of the two-digit numbers $a7$ and $b2$ drops by 150 if you swap the tens digits.

Part 1: What are the tens digits? Find one answer.

Part 2: Now find as many answers as you can. What patterns do you see?

Part 3: What happens with other combinations of tens digits?

Name _____

Problem 3: Testing the Waters

Part 1: How does the product 17×3 change when you exchange the ones digits to get 13×7 ?

Part 2: Which other ones digits cause the same change? What patterns do you see?

Part 3: What happens with other combinations ones digits?

Name _____

Problem 3: Diving Deeper

Choose one or more questions to answer, or think of your own new questions to explore.

1. What happens if $a < b$?
2. What happens if you change the numbers to $a6$ and $b2$?
3. How can you use the difference between the ones digits to predict the differences between the products?
4. What causes all of these patterns? Can you prove them?

Problem 3 Teacher's Guide

Topics

Multi-digit multiplication; patterns; algebraic expressions

Materials

None

The grade level notes below are a rough guide for working with different age groups. Read through all three grades to get a feel for the likely range of students' thinking. Their approaches may vary a lot, so never feel limited to a particular grade level. Observe your students' ideas and strategies to assess their thinking, and build from there.

Grade 4 Common Core Standards: 4.OA.C.5 4.NBT.A.1 4.NBT.B

If students have trouble multiplying or they spend too much time calculating, consider beginning with Testing the Waters. Students may need help learning to organize their data (probably in tables) on their thinking paper. This is important, because it will help them find patterns.

Grade 5 Common Core Standards: 5.OA.A 5.NBT.B.5

Fifth graders may still need some help learning to organize their data. Encourage them to try using algebraic expressions to describe their discoveries. For example, the difference in the products is $50 \cdot (b - a)$.

Grade 6 Common Core Standards: 6.EE.A

If you have students who know enough about the distributive property and manipulating algebraic expressions, ask them to prove their discoveries. For example,

$$\begin{aligned} (10a + 7)(10b + 2) - (10b + 7)(10a + 2) &= \\ (100ab + 20a + 70b + 14) - (100ab + 20b + 70a + 14) &= \\ 100ab - 100ab + 20a - 70a + 70b - 20b + 14 - 14 &= \\ -50a + 50b = 50b - 50a &= \\ 50(b - a) \end{aligned}$$

proves that the difference of the products is fifty times the difference of the tens digits.

Solutions begin on page 41.

Name _____

Problem 4

Imagine a planet with insects called zzubz that build honeycombs from hexagons just like bees on Earth, but their hexagons are not *regular*. The *interior* angles are congruent, but the sides have three different lengths.

Part 1: Draw an example of a zzub's hexagon.

Part 2: Identify any parallel or perpendicular sides.

Part 3: Can you find any lines of symmetry? Other symmetry?

Part 4: Make a honeycomb from your hexagon.

Name _____

Problem 4: Testing the Waters

Imagine a planet with insects called zzubs that build honeycombs from hexagons just like bees on Earth, but the hexagons are not *regular*. The *interior* angles are congruent, but the sides have three different lengths. Use pattern blocks or triangle dot paper to:

Part 1: Make a regular hexagon from equilateral triangles.

Part 2: Make an example of a zzub's hexagon from equilateral triangles.

Name _____

Problem 4: *Diving Deeper*

Choose one or more questions to answer, or think of your own new questions to explore.

1. How many different hexagons like this (in which the angles are congruent but some sides are not) can you make?
2. Do all of your hexagons have the same pattern of parallel lines?
3. Do your hexagons have lines of symmetry?
4. Do your hexagons have other kinds of symmetry?
5. Does your entire honeycomb have any symmetry? If so, are its symmetries the same as those of the individual hexagons?
6. Can you create heptagons or octagons whose angles are all congruent but whose sides are not? Can you make 'honeycombs' from them?

Problem 4 Teacher's Guide

Topics

Polygons; angles; parallel and perpendicular lines; reflections, rotations, and symmetry; tessellations

Materials

Protractor and ruler (needed); compass (optional)
Triangle dot paper on page 59 (for Testing the Waters)

The grade level notes below are a rough guide for working with different age groups. Read through all three grades to get a feel for the likely range of students' thinking. Their approaches may vary a lot, so never feel limited to a particular grade level. Observe your students' ideas and strategies to assess their thinking, and build from there.

Grade 4 Common Core Standards: 4.OA.C.5 4.MD.C 4.G.A

Introduce or have students research the terms *regular* polygon and *interior* angle if needed. This problem may be especially challenging for younger students. Consider using Testing the Waters in place of Part 1. Otherwise, give students the tools from the Materials section, and tell them that they may decide which tools to use (if any) and how to apply them to the problem.

Grade 5 Common Core Standards: --- (This problem extends work on angles begun in grade 4.)

Give students time to try to solve the problem with the tools in the Materials section. Some of them may be able to use their knowledge of angles to find solutions with only these tools. If they don't make progress, use Testing the Waters. In Part 3, they may be surprised to discover that there are no lines of symmetry! See the Solutions for more information about symmetry.

Grade 6 Common Core Standards: --- (This problem anticipates work in grade 7.)

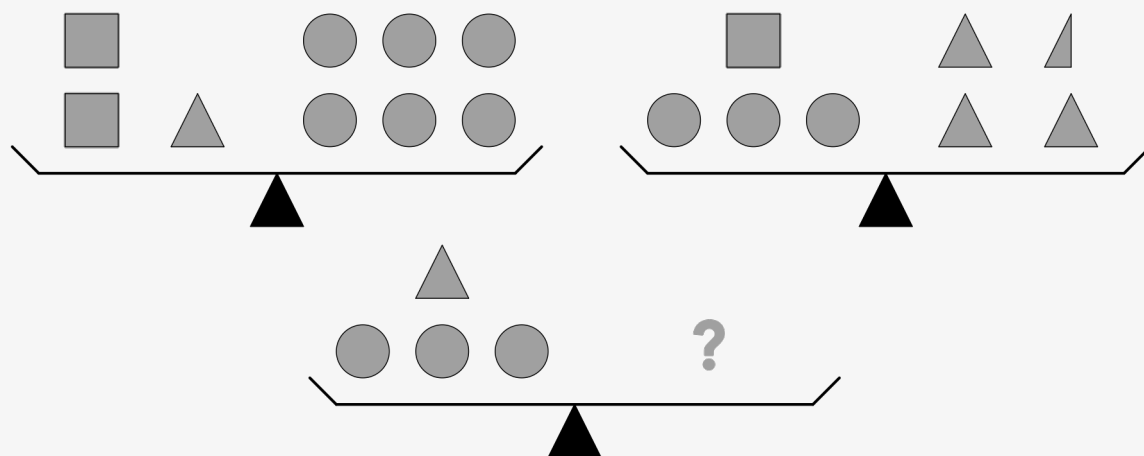
Older students may apply the fact that regular hexagons have interior angles measuring 120° . If students have extra time, have them try the Diving Deeper questions.

Solutions begin on page 43.

Name _____

Problem 5

What is needed on the right side of the bottom scale in order to balance it*?

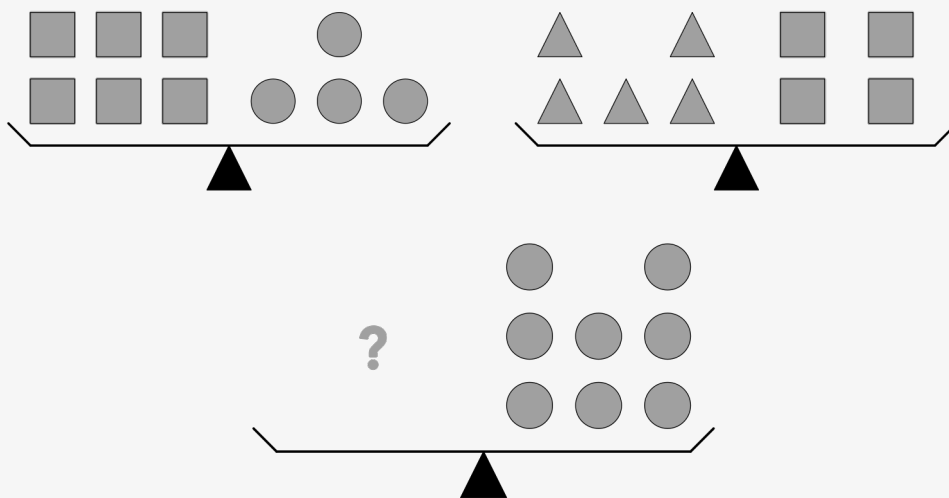


* These scales are balanced when both sides have the same weight. In a real scale, objects that are farther from the center have a stronger effect!

Name _____

Problem 5: Testing the Waters

How many triangles are needed to balance the eight circles*?



* These scales are balanced when both sides have the same weight. In a real scale, shapes that are farther from the center have a stronger effect!

Name _____

Problem 5: Diving Deeper

Choose one or more questions to answer, or think of your own new questions to explore.

1. Write an algebraic equation for each scale, including your solution(s).
2. What numbers could T, C, and S stand for?
3. Create your own scale problem. Trade problems with another person and solve them.
4. Make a table, formula, and graph for each pair of variables showing the relationship between them.
5. Why are there so many possible sets of values of T, C, and S? Is it possible to change the problem so that there is only one set of values (for instance, so that T, C, and S *must* equal 6, 4, and 9)?

Problem 5

Teacher's Guide

Topics

Algebraic expressions and equations; proportional reasoning

Materials

None

The grade level notes below are a rough guide for working with different age groups. Read through all three grades to get a feel for the likely range of students' thinking. Their approaches may vary a lot, so never feel limited to a particular grade level. Observe your students' ideas and strategies to assess their thinking, and build from there.

Grade 4 Common Core Standards: 4.OA.C

If students ask what kinds of shapes should be on the right side, suggest that they choose one shape and try to figure out how many would be needed to balance the scale. If they have trouble getting started, use Testing the Waters. Ask students how they can keep the scale balanced even while they change the shapes on it. See the 'Strategies for keeping the scales balanced' in the Solutions.

Some students may develop 'codes' in order to save time drawing. For example, they may show three circles as 3C. You may even suggest this, but don't press for it. Some may need to see the pictures.

Grade 5 Common Core Standards: 5.OA.A

Challenge students to find many solutions. They may begin to think of their algebraic representations in new ways. For example, they may realize that each shape can represent a particular number, enabling them to interpret an expression like '3C' not just as 'three circles' but as '3 times C.' Students who are thinking this way may want to work on some of the Diving Deeper questions.

Grade 6 Common Core Standards: 6.EE.A 6.EE.B

If students are very comfortable representing their ideas with variables, encourage them to show their entire thinking process using algebraic equations and to explain how they created each new equation from the previous one. The Diving Deeper questions may be appropriate for many sixth graders.

Solutions begin on page 45.

*The word *unique* means that there is only one solution, not that there is anything else special about it.

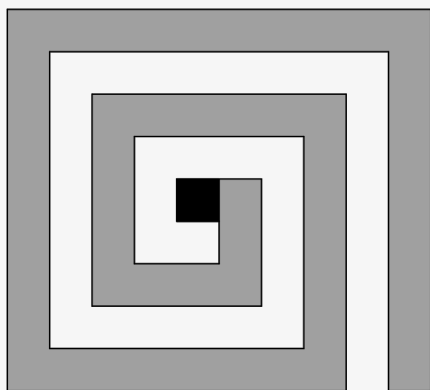
Name _____

Problem 6

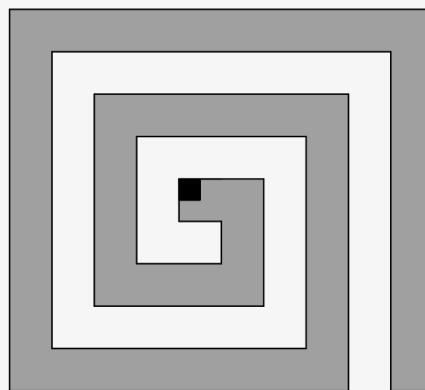
If the black square is one square unit, find the area and perimeter of each grey spiral.

- Explain your strategies, especially any patterns that you use.
- How do the area and perimeter numbers change from Part 1 to Part 2? Why?

Part 1



Part 2



Problem 6: Testing the Waters

There is no separate handout for Testing the Waters on this problem. This page includes a couple of suggestions for supporting students in solving the original problem.

Part 1:

If students need help getting started, suggest that they draw a grid on top of the spiral. (The black square will be one of the squares in the grid.) They won't need to add much to the picture, because the border of the spiral already includes a lot of the grid!

Part 2:

Each square on the grid will be divided into four squares.

Name _____

Problem 6: Diving Deeper

Choose one or more questions to answer, or think of your own new questions to explore.

1. Why do the area and perimeter change *differently* when you make the square unit smaller?
2. Experiment with continuing to make the square unit even smaller. What patterns do you see?
3. Experiment with making the square unit larger. What patterns do you see? How do they related to the patterns in question 2?
4. If you haven't already, find a way to make the area number become greater than the perimeter number. How does it make sense that the relationship between these two numbers can change when the shape stays the same?
5. Extend the spiral a little bit at a time. (Graph paper will be helpful for this.) Look for patterns in the way that the area and perimeter numbers increase.

Problem 6

Teacher's Guide

Topics

Area, perimeter, and their relationship; patterns

Materials

None

The grade level notes below are a rough guide for working with different age groups. Read through all three grades to get a feel for the likely range of students' thinking. Their approaches may vary a lot, so never feel limited to a particular grade level. Observe your students' ideas and strategies to assess their thinking, and build from there.

Grade 4 Common Core Standards: 4.MD.A.3

If students have trouble getting started, see the Testing the Waters page. Watch to see if they look for patterns. If not, you may ask if any of them can see consecutive numbers in the spiral.

Grade 5 Common Core Standards: 5.OA.A 5.OA.B

Some students may notice two spirals—a white and a grey one. Since the white one is smaller, they may begin with the area of the rectangle containing both spirals and subtract the area of the white one. See Strategy #3 in the Solutions.

Grade 6 Common Core Standards: 6.EE.A.2.b 6.G.A.1

If students find the answers quickly, ask them to focus on describing and justifying patterns, and have them work on some of the Diving Deeper questions.

Solutions begin on page 47.

Name _____

Problem 7

Juan and Julia are doing a science experiment. They drop a ball and set a camera to snap a photo of it every $\frac{1}{4}$ of a second. They join the photos into one image (page 29). By studying their photo, they discover that after 1 second, the ball has dropped 16 feet.

Part 1: How far has the ball dropped after 1.25 seconds?

Part 2: How far will the ball have dropped after 5.75 seconds?

Part 3: How long would it take for the ball to drop 1444 feet?

Part 4: Write a formula that tells you how far the ball drops after t seconds.

Problem 7: Testing the Waters

There is no separate handout for Testing the Waters on this problem. This page includes a couple of suggestions for supporting students in solving the original problem.

- If students are having difficulty seeing patterns in their measurements, point out that it is not the *actual* lengths on the photo that matter, but the relationships between the lengths.
- If they need further support, suggest the second or third strategy under ‘Strategies to measure distances between dots’ in the Solutions.

Name _____

Problem 7: Diving Deeper

Choose one or more questions to answer, or think of your own new questions to explore.

1. Create a graph showing the relationship between t and D .
2. About how long would it take for the ball to drop one mile?
3. How confident are you that your answer to question 2 is an accurate prediction of the time it would actually take a ball to drop one mile? Explain.
4. Write a formula that tells you how long it takes the ball to drop D feet.

Problem 7 Teacher's Guide

Topics

Decimal number sense; measuring lengths; analyzing and extending patterns; tables and algebraic equations

Materials

Problem 7 handout (Diagram of Juan and Julia's photo on page 60); ruler (if students request it)

The grade level notes below are a rough guide for working with different age groups. Read through all three grades to get a feel for the likely range of students' thinking. Their approaches may vary a lot, so never feel limited to a particular grade level. Observe your students' ideas and strategies to assess their thinking, and build from there.

Grade 4 Common Core Standards: 4.OA.A 4.OA.C 4.NBT.B 4.NF.C

Do not offer rulers, but provide one if students ask for it. If students can't see patterns in their numbers because they have measured in inches or centimeters, suggest that they try to think of a different measurement strategy. See Testing the Waters.

If students have trouble seeing the patterns because they are not organizing their work, guide them in setting up a table.

Grade 5 Common Core Standards: 5.OA.A 5.OA.B 5.G.A

Grade 5 students may need similar types of support as younger students. They may be more able to discover patterns in the numbers even if they measured in inches or centimeters. It may also be a little easier for them to spot patterns involving multiplication. This, in turn, may help them to progress to Parts 3 and 4.

Grade 6 Common Core Standards: 6.EE.A 6.EE.B 6.EE.C

Sixth graders may be more likely to reach Part 4. If they have time, ask them to come up with multiple equations and to explain why each equation always gives the same answer. Once they have one or more equations, they will be in a good position to tackle the Diving Deeper questions.

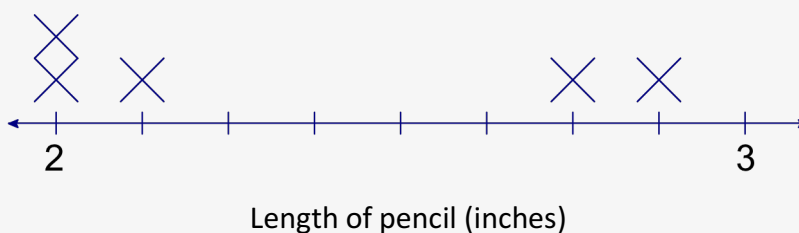
Solutions begin on page 49.

Name _____

Problem 8

Eight students in Ms. Blanding's class like to use their pencils until they get very short. When they measured their pencils to the nearest eighth of an inch, all of the lengths were between 2 inches and 3 inches *inclusive**. The *mean* length was $2\frac{3}{8}$ inches, which was $\frac{1}{4}$ of an inch greater than the *median* length.

Finish the line plot. Explain your thinking.



Explanation:

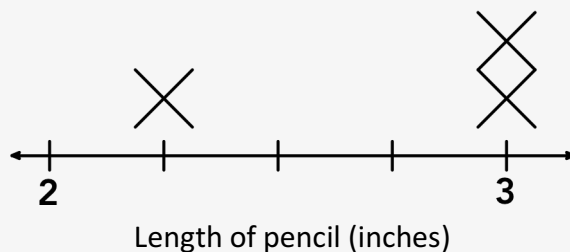
**Inclusive* means that the numbers 2 and 3 are included.

Name _____

Problem 8: Testing the Waters

Four students in Ms. Blanding's class like to use their pencils until they get very short. When they measured their pencils to the nearest quarter of an inch, all of the lengths were between 2 inches and 3 inches *inclusive**. The *mean* length was $2\frac{3}{4}$ inches.

Finish the line plot. Explain your thinking.



**Inclusive* means that the numbers 2 and 3 are included.

Name _____

Problem 8: Diving Deeper

Choose one or more questions to answer, or think of your own new questions to explore.

1. Keep the original problem except for two things: (1) Change the mean and median as shown below, and (2) Create a new line plot from scratch. Explain your thinking.

mean: $2\frac{1}{2}$

median: $2\frac{5}{16}$

2. What is the mode for your data? Could there be a solution with a different mode?

Problem 8

Teacher's Guide

Topics

Visualizing fractions on a number line; adding fractions; line plots; mean and median

Materials

None

The grade level notes below are a rough guide for working with different age groups. Read through all three grades to get a feel for the likely range of students' thinking. Their approaches may vary a lot, so never feel limited to a particular grade level. Observe your students' ideas and strategies to assess their thinking, and build from there.

Grade 4 Common Core Standards: 4.NF.A 4.NF.B.3 4.NF.B.4 4.MD.B.4

Ensure that your students understand the definitions of *mean* and *median*. If they are not yet familiar with line plots, show them the plot and give them time to think about it. They may be able to figure out what it means on their own.

Problems like this are especially beneficial for students who have not yet learned to add fractions with unlike denominators. Instead of pre-teaching procedures for this, ask them to use the number line to help them figure out what they need to know. For example, when they try to determine the median, ask them to visualize $\frac{1}{4}$ less than $2\frac{3}{8}$ on the line. If they are unable to make a mean of $2\frac{3}{8}$, have them begin with the Testing the Waters question.

Grade 5 Common Core Standards: 5.NF.A 5.NF.B.4.a 5.MD.B

Some students may develop clever strategies for managing the fractions and mixed numbers. For example, they may ignore the 'eighths' units and focus only on the numerators of the fractions when doing calculations! (They would then be dealing with whole numbers between 16 and 24 instead of mixed numbers between 2 and 3.) Other students may temporarily ignore the whole number part of the mixed numbers when doing their calculations. (They would then be dealing with 'eighths' fractions between 0 and 1.)

Grade 6 Common Core Standards: 6.SP.A.3 6.SP.B.4

Students who finish the problem more quickly than others should be sure to explain their thinking completely and clearly. Then they may work on Diving Deeper questions.

Solutions begin on page 51.

Intrepid Math Solutions

Problem 1

Maria has completed 7 puzzles. Each puzzle has 19 rows and 61 columns of pieces.

Basic strategies

- Divide to search for small factors of 8113 (using any methods that make sense).
- Realize that 2 cannot be a factor, since 8113 is odd. (This extends to all even numbers.)
- Realize that 5 cannot be a factor, since 8113 does not end in 5 or 0. (This extends to all multiples of 5.)
- Once you discover that 7 is a factor, work with 1159 ($8113 \div 7$) instead of 8113.
- Notice that one factor is quite a bit smaller than the other. (The puzzle is “long and thin.”)
- Multiply and divide (using any method) in order to test potential factors.
- Use estimation to help you choose and test numbers.

Intermediate strategies

Use the basic strategies along with others to make the process more efficient:

- Discover that since 1159 has a ones digit of 9, the ones digits of its factors must be 1 and 9, 3 and 3, or 7 and 7.
- Realize that one factor must be less than 40. ($40 \cdot 40 = 1600$, which is too large.)
- Recognize that a number in the upper 30s will not be the smaller factor (too close to 40).
- Put these facts together to limit the numbers you have to test as potential factors:
11, 13, 17, 19, 21, 23, 27, 29, 31, 33.

Advanced strategies

Use basic and intermediate strategies along with others to make the process more efficient:

- Notice that since 3 is not a factor (of either 8113 or 1159), no multiple of 3 can be a factor. This eliminates 21, 27, and 33 from consideration.
- See from the picture that the width is about 3 times the length. Estimate to test numbers.

Testing 11:	$11 \times 3 = 33$	$11 \times 33 \approx 10 \times 30 = 300$ is too small.
Testing 13:	$13 \times 3 = 39$	$13 \times 39 \approx 10 \times 40 = 400$ is too small.
Testing 17:	$17 \times 3 = 51$	$17 \times 51 \approx 18 \times 50 = 900$ is too small.
Testing 19:	$19 \times 3 = 57$	$19 \times 57 \approx 20 \times 60 \approx 1200$ is close!
Testing 23:	$23 \times 3 = 69$	$23 \times 69 \approx 20 \times 70 = 1400$ is too big.

This suggests that 19 is a factor, which is correct! $8113 \div 19 = 61$.

Problem 1: Testing the Waters

Maria has completed 5 puzzles.

Each puzzle has 11 rows and 13 columns (or 13 rows and 11 columns) of pieces.

Problem 1: Diving Deeper

1. The problem has only one realistic solution, because the factors are all prime numbers. It is possible that one of the factors could be 1, but a puzzle with 1 row or column is not realistic, and a *collection* of puzzles should have more than 1 puzzle.
2. Students could create more problems from numbers with three prime factors, or they may find ways that make all but one of the solutions unrealistic.
3. Students may use prime factorizations to control the number of factors. Numbers with fewer prime factors and/or repeated prime factors will have fewer factors overall than numbers with many different prime factors.

Problem 2

Part 1

Flora lands on 0.090909. She keeps getting closer to $\frac{1}{11}$, but she never quite gets there.

If students can't figure out what fraction she is getting closer to, they may still be able to estimate that the number is a little less than 1 tenth.

Suggestion

Instead of using an algorithm to add and subtract decimals, encourage students to think about place value and to *visualize* the distances. Use number and word notation to support this kind of thinking. For example: Write 0.1 as '1 tenth.' Think of 'tenth' as a unit, and picture its size on the number line.

A place value strategy for Part 1

1 tenth = 10 hundredths. 1 hundredth less than 10 hundredths is 9 hundredths. (0.09)

9 hundredths = 90 thousandths. 1 thousandth greater than 90 thousandths is 91 thousandths. (0.091)

91 thousandths = 910 ten-thousandths. 1 ten-thousandth less than 910 ten-thousandths is 909 ten-thousandths. (0.0909), etc.

Part 2

Flora lands on 0.984375. She keeps getting closer to 1, but she never quite gets there.

($0.984375 = \frac{63}{64}$. Some students may do the problem with fractions as well as decimals.)

A place value strategy for Part 2

5 tenths = 50 hundredths. (0.5)

Half of 50 hundredths is 25 hundredths.

50 hundredths + 25 hundredths = 75 hundredths (0.75)

75 hundredths = 750 thousandths. 25 hundredths = 250 thousandths.

Half of 250 thousandths = 125 thousandths.

750 thousandths + 125 thousandths = 875 thousandths (0.875)

etc.

Problem 2: Testing the Waters

Flora will land on 0.111111.

If she continues, she will keep moving to the right more and more slowly.

She will get closer and closer to $\frac{1}{9}$ but will never quite get there.

Students, especially younger ones, are not expected to know what fraction she is getting closer to, but some may guess by looking at a number line and estimating or by noticing that $0.111111 \cdot 9 = 0.999999$ (which is obviously very close to 1)!

Problem 2: Diving Deeper

1. 0.888888.

Flora is getting closer and closer to $\frac{8}{9}$.

2. 0.909090.

Flora is getting closer and closer to $\frac{10}{11}$.

3. 0.328125.

The decimal equals $\frac{21}{64}$. Flora is getting closer to 0.333333..., or $\frac{1}{3}$.

Problem 3

Part 1

See Part 2 for possible answers. Students need only one of these for Part 1.

Part 2

Answers

$$a = 1 \text{ and } b = 4$$

$$a = 2 \text{ and } b = 5$$

$$a = 3 \text{ and } b = 6$$

$$a = 4 \text{ and } b = 7$$

$$a = 5 \text{ and } b = 8$$

$$a = 6 \text{ and } b = 9$$

Note: $a = 0$ and $b = 3$ is a solution if you think of 07 as 7 and 02 as 2.

Some patterns

b is always 3 greater than a .

The difference of the products is fifty times the difference of the tens digits.

Part 3

The possible amounts of decrease are multiples of 50.

0, 50, 100, 150, 200, 250, 300, 350, and 400.

The difference of the products is *always* fifty times the difference of the tens digits!

Suggestion

Encourage students to organize their data. If they need help, suggest using input/output tables.
(There will be *two* inputs for each output!)

Problem 3: Testing the Waters

Part 1: The product increases by 40 (from 51 to 91).

Part 2: The other pairs of ones digits that make the product increase by 40 are:

4,0; 5,1; 6,2; 8,4; and 9,5

Pattern: The ones digit of the two-digit number is always 4 greater than in the one-digit number.

Part 3: When the tens digit is 1, the difference of the products is always 10 times the difference of the ones digits.

Part 4: When the tens digit is 2, the difference of the products is always 20 times the difference of the ones digits.

When the tens digit is 3, the difference of the products is always 30 times the difference of the ones digits, etc.

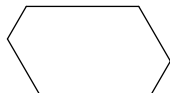
Problem 3: Diving Deeper

1. If $a < b$, then the product will increase instead of decrease when you swap the digits.
2. If you change the numbers to $a6$ and $b2$, the possible differences will be multiples of 40 instead of 50.
3. The possible differences in the products are 10 times the difference in the ones digits.
4. Answers will vary. Some students may be able to give algebraic justifications. Others may reason using properties of numbers and operations.

Problem 4

Part 1

A sample hexagon



Some students may observe that they could have created this type of hexagon by starting with a certain parallelogram (containing pairs of 60° and 120° angles) and 'chopping off' opposite corners with parallel cuts!

Part 2

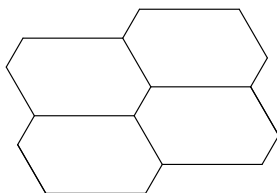
Opposite sides of the hexagon are parallel. There are no perpendicular sides.

Part 3

There are no lines of symmetry! (See 'Reflection Symmetry' below.)

There is half-turn (rotation) symmetry around the center of the hexagon.

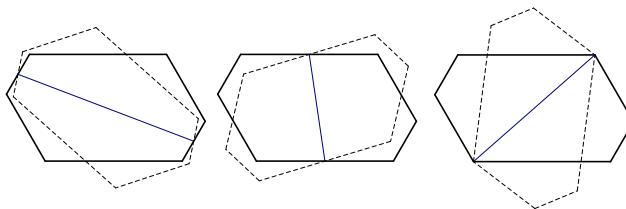
Part 4



Note: The honeycomb is a *tessellation*: a collection of shapes that cover an endless flat surface in a repeating pattern with no gaps or overlaps. Students may create or research examples of other tessellations and explore their symmetries.

The lack of reflection symmetry

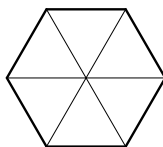
Here are some examples of what happens if you reflect the hexagon over lines that you might think would be lines of symmetry. It may be an interesting challenge for some students to try drawing reflections like these!



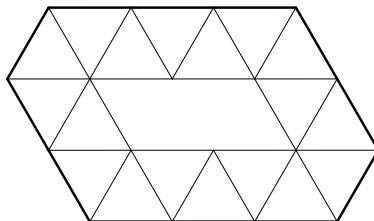
These lines of reflection are not lines of symmetry, because each hexagon looks different after you reflect it.

Problem 4: Testing the Waters

Part 1:



Part 2:



Note: It is optional to fill the rest of the hexagon with triangles.

Problem 4: Diving Deeper

1. You can make an infinite number of different hexagons like this. As long as you keep the opposite sides congruent, you make the lengths anything you like.
2. Yes, the opposite sides of all of the hexagons should be parallel.
3. If the hexagons have three different side lengths as stated in the problem, there will be no lines of symmetry. If they have only two different side lengths, there will be two lines of symmetry.
4. All of the hexagons should have *half-turn symmetry*.
5. The honeycomb still has only half-turn symmetry, but (imagining that it continues forever) it now has four different centers of rotation. Can you find them?
6. It is not possible to create heptagons whose angles are all congruent but whose sides are not, because they have an odd number of sides. With octagons and other polygons that have an even number of sides, you can do this.

Problem 5

Solutions involving one shape:

(1) 3 triangles

(2) 2 squares

(3) 4.5 circles

Solutions involving multiple shapes:

(1) 1 circle and $2\frac{1}{3}$ triangles

(2) 2 circles and $1\frac{2}{3}$ triangles

(3) 3 circles and 1 triangle (of course!)

(4) 4 circles and $\frac{1}{3}$ of a triangle

(5) 1 square and 1.5 triangles

(6) 2 triangles and 1.5 circles

There are *many* more solutions and many strategies for finding them.

Strategies for keeping the scales balanced

- Add the same shapes to both sides of the scale.
- Remove the same shape from both sides of the scale.
- Double, triple, take half, take a third, etc. of each side of the scale.
- Replace a set of shapes on one side of the scale by something else that balances it.

Example of a solution process

- | | |
|--|----------------|
| 1. Double both sides of the right scale. | $6C + 2S = 7T$ |
| 2. Replace the 6 circles by 2 squares and 1 triangle.
(The left scale shows that these combinations balance.) | $4S + 1T = 7T$ |
| 3. Remove 1 triangle from both sides the scale. | $4S = 6T$ |
| 4. Take half of both sides of the scale. | $2S = 3T$ |
| 5. Go back to the left scale. | $2S + 1T = 6C$ |
| 6. Replace 2 squares by 3 triangles. (See step 4.) | $4T = 6C$ |
| 7. Take half of both sides of the scale. | $2T = 3C$ |
| 8. Go to the bottom scale. | $3C + 1T$ |
| 9. Replace 3 circles by 2 triangles. (See step 7.) | $3T$ |

Conclusion: **3 triangles** will balance the 3 circles and 1 triangle. (See the first solution listed.)

Problem 5: Testing the Waters

15 triangles

Problem 5: Diving Deeper

1. Left scale: $2S + T = 6C$ Right scale: $3C + S = 3.5T$ Bottom scale: $3C + T = ?$
Students may insert their solution(s) in place of the question mark.
2. The simplest answer is probably $T = 6$; $C = 4$; $S = 9$.
To get other answers, just multiply 6, 4, and 9 by the same number.
4. Each pair of variables has a clearly defined relationship, and there are many ways to express it: $2T = 3C$; $T = 1.5C$, $C = (2/3)T$, etc. Students' graphs should be straight lines through $(0, 0)$ whose steepness (slope) is determined by how quickly one variable changes compared to the other.
5. Since the scales contain only shapes (not numbers), they express relationships *between* variables, but not specific values of the variables.

Problem 6

Part 1	Area: 54 square units	Perimeter: 110 units
Part 2	Area: 216 square units	Perimeter: 220 units

The perimeter number becomes 2 times greater in Part 2, because each side of a unit square from Part 1 is split into two parts.

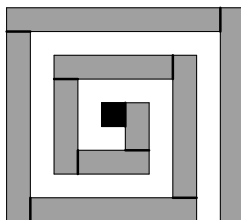
The area number becomes 4 times greater in Part 2, because each unit square from Part 1 is split into 4 unit squares.

Some strategies for calculating area

#1 Notice patterns of consecutive numbers.

For example, in Part 1, the grey area is

$$(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) + 9 = 45 + 9 = 54.$$



The perimeter calculation involves *two* sets of consecutive number sums.

You could calculate the area for Part 2 using sums of consecutive multiples of 4.

#2 Use the commutative and associative properties to add consecutive numbers.

For instance:

$$\begin{aligned} 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 &= \\ (1 + 9) + (2 + 8) + (3 + 7) + (4 + 6) + 5 &= \\ 10 \cdot 4 + 5 &= 45 \end{aligned}$$

#3 Notice the white “spiral within a spiral.”

It is smaller and follows a slightly simpler pattern. Subtract its area from the area of the rectangle containing both the white and grey spirals.

$$(10 \cdot 9) - (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8) = 90 - 36 = 54$$

Problem 6: Diving Deeper

1. The area and perimeter can change *differently* because they are different kinds of measurements. However, there is a relationship between the ways that the two of them change. (See the answers to question 2.)
2. The area numbers will continue to increase more quickly than the perimeter numbers. Some students may notice that when the perimeter increases by a certain factor, the area increases by the square of that factor.
3. The patterns that you see in problem 2 are reversed. The area decreases more quickly than the perimeter. If students continue for a while, they will begin dealing with fractional amounts.
4. The earlier questions should help students do this. The relationship between the area and perimeter numbers can change because length and area are different kinds of units. For example, it does not make any more sense to ask if the area is greater than the perimeter than it does to ask if 3 miles is greater than 2 hours.

Problem 7

Part 1: 25 feet

Part 2: 529 feet

Part 3: 9.5 seconds

Part 4: $D = (4 \cdot t) \cdot (4 \cdot t)$

$$D = (4 \cdot t)^2$$

$$D = 16 \cdot t \cdot t$$

$$D = 16 \cdot (t^2)$$

Notes

These are correct formulas for the describing the motion of falling objects on Earth! Each of the formulas is *equivalent*. (Each gives the same value of D for any t that you choose.) Younger students may need to use a calculator to verify certain formulas for decimal values.

Strategies to measure distances between dots

- Measure each of them with a ruler. (Note: This may make it hard to see the patterns.)
- Split the distance between 0 and 1 seconds into halves, fourths, eighths, and sixteenths.
- Use the distance between 0 and 0.25 seconds as a unit. Mark off multiples of this length between the dots.

Suggestion

It may help to organize measurements in a table.

t (sec)	0	0.25	0.5	0.75	1	1.25
D (ft)	0	1	4	9	16	25

Observations

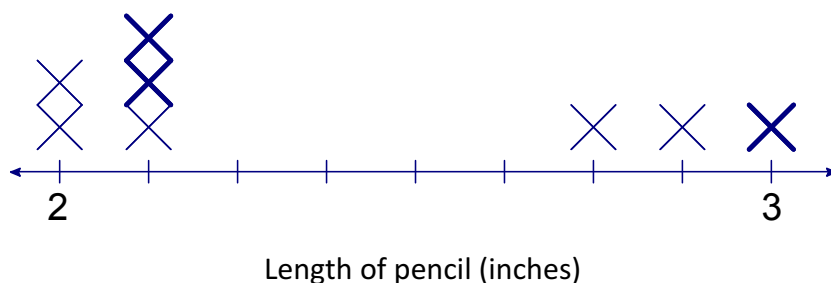
- D increases by consecutive odd numbers: $0 + 1 = 1$ $1 + 3 = 4$ $4 + 5 = 9$ $9 + 7 = 16$
- The values of D are *square numbers*: $1 \cdot 1$, $2 \cdot 2$, $3 \cdot 3$, $4 \cdot 4$, etc.

Either pattern can help students make predictions. The second pattern is generally easier to use when numbers get larger.

Problem 7: Diving Deeper

1. If t is on the horizontal axis, the graph should start at $(0, 0)$ and curve upward more and more steeply. If D is on the horizontal axis, the graph will curve upward more and more slowly instead. (t would traditionally be on the horizontal axis if you are thinking of the distance as depending on the time.)
2. It would take a little more than 18 seconds for the ball to drop one mile. Some students may extend their tables until they find the answer. Others may 'guess, test, and revise,' possibly using their table to guide their guesses. Still others may use one of their algebraic formulas and 'work backwards.'
3. It is good to be skeptical about predictions for numbers that are far away from the measured data. In this case, the faster the ball goes, the greater the air resistance becomes, and the less accurate the formula becomes.
4. $t = \sqrt{D \div 16}$ This formula basically describes the 'working backwards' strategy that some students may have discovered in question 2.

Problem 8



The bold Xs are the ones that students need to fill in.

Important ideas

You need the two Xs above $2\frac{1}{8}$ in order to make the median be $2\frac{1}{8}$.

In order to get a mean of $2\frac{3}{8}$, you can think of $\underline{\hspace{1cm}} \div 8 = 2\frac{3}{8}$. There are many ways to figure out what belongs in the blank. For example:

- You can realize that you need to get 2 R3.
- You can multiply $2\frac{3}{8}$ by 8 (or add it 8 times) to get $\frac{19}{8}$.
- You can think of $2\frac{3}{8}$ as an improper fraction, $\frac{19}{8}$.

It may be helpful for some students to think of $\frac{19}{8}$ as '19 eighths,' imagining 'eighths' as a unit. This may enable some of them to make the connection between $\frac{19}{8}$ and $19 \div 8$.

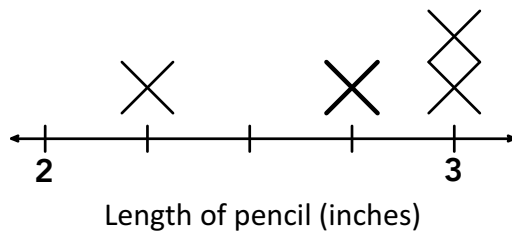
Any way you look at it, 19 belongs in the blank. The other seven numbers add up to 16, so the last number has to be 3 so that they will have a sum of 19.

The possibility of other solutions

You could move one of the Xs from $2\frac{1}{8}$ to 2 without affecting the median. However, this would decrease the mean. To bring the sum of the eight numbers back to 19, you would need to increase another of the values by $\frac{1}{8}$. The only other of the three numbers that could be increased without changing the median is 3. This is not possible since all of the values are between 2 and 3.

Problem 8: Testing the Waters

The bold **X** above $2\frac{3}{4}$ is the one that students need to add. There are many possible thinking strategies.



Problem 8: Diving Deeper

1. The line plot must have five Xs above $2\frac{1}{2}$ and three Xs above 2. Anything else with a median of $2\frac{1}{2}$ would have too high a mean.
2. The mode is $2\frac{1}{2}$ because it is the value that appears most often. No other mode is possible, because there is no other solution to the problem.

Triangle dot paper for Problem 4a

