Intrepid Math

Challenging Common Core Problems

Grades 4 – 6
Set 1

from 5280 math
by Jerry Burkhart

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Intrepid Math
Challenging Common Core Problems
Quick Start Guide

1. **Prepare**
   Choose one problem.
   Make copies.
   Gather materials.
   Familiarize yourself with the problem.

2. **Introduce**
   Distribute the problem, materials, and *thinking paper**.
   Give directions.
   Establish expectations.
   Set an estimated time frame.

3. **Support**
   Facilitate discussion or check in with individuals and groups.
   Help students clarify their thoughts.
   Ask guiding questions.
   Encourage and motivate students through the tough spots.
   Determine when students have reached a stopping point.

4. **Wrap up**
   Gather completed work with explanations.
   Acknowledge students' effort and progress.
   Summarize what was learned.
   Assess student work.

Advice for students
Plan on spending a few days or more on many of the problems.
The problems are challenging. Expect to get stuck and to make some mistakes.
Take your time. It is more important to learn new things than to finish the entire problem.

*See the FAQs to learn about thinking paper and strategies for teaching with the problems.

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Name ________________________________

Problem 1

Staci chooses a three-digit number greater than 500.
She writes the number backwards and then subtracts the two numbers.

Part 1: The difference is 792. What number did she choose?
Part 2: The difference is 0. What number did she choose?
Part 3: The difference is 396. What number did she choose?
Part 4: What other differences are possible? What patterns do you see?

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Name ________________________________

Problem 1: *Testing the Waters*

Staci chooses a two-digit number greater than 50.
She writes the number backwards and then subtracts the two numbers.

Part 1: The difference is 72. What number did she choose?
Part 2: The difference is 0. What number did she choose?
Part 3: The difference is 36. What number did she choose?
Part 4: What other differences are possible? What patterns do you see?

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Problem 1: *Diving Deeper*

Choose one or more questions to answer, or think of your own new questions to explore.

1. Why do you think the problem limits Staci's choices to numbers greater than 500? Was this necessary?
2. What happens with four-digit numbers whose two middle digits are equal (for example, 8661)?
3. What if the two middle digits are not equal?
4. Can you find anything interesting about numbers with five or more digits?
5. *Why* are the differences for the original problem always multiples of 99?

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Problem 1
Teacher's Guide

Topics
Subtraction; patterns; multiples

Materials
None

The grade level notes below are a rough guide for working with different age groups. Read through all three grades to get a feel for the likely range of students' thinking. Their approaches may vary a lot, so never feel limited to a particular grade level. Observe your students' ideas and strategies to assess their thinking, and build from there.

Grade 4 Common Core Standards: 4.NBT.A 4.OA.B.4
This is a good problem for early in the school year. There is a lot to explore, but it can be solved with concepts from earlier grades. Students learn about problems that have more than one answer. They also get used to spending a long time on one problem and using thinking paper to record their ideas. There are many patterns, and they are easier to discover when students write their ideas down.

Grade 5 Common Core Standards: 5.OA.B
Grade 5 students may get a little further with the problem and observe more patterns. They are also more likely to pursue the Diving Deeper questions.

Grade 6 Common Core Standards: 6.NS.B 6.EE.A
Advanced students who know enough algebra may be able to show what causes the patterns by writing and simplifying algebraic expressions. For example, when the hundreds digit is 3 greater than the ones digit, the difference between the number *abc* and its "reversal" *cba* is

$$(100a + 10b + a - 3) - (100(a - 3) + 10b + a).$$

Subtracting and applying the distributive property leads to the answer 297! Replacing the number 3 by a variable such as n gives the answer $99n$, which shows why every answer is a multiple of 99!

Solutions begin on page 37.

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Name ______________________________

Problem 2

Draw a polygon that contains

Part 1: Exactly one reflex, one obtuse, one right, and one acute angle.

Part 2: One *or more* of each of these types of angles.

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Name ________________________________

Problem 2: *Testing the Waters*

Draw a triangle that contains

Part 1: A right angle and an obtuse angle.

Part 2: An obtuse angle and two acute angles.

Part 3: A reflex angle.

What combinations of angles are possible? Why?

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Name ______________________________

Problem 2: *Diving Deeper*

Choose one or more questions to answer, or think of your own new questions to explore.

Keep exploring new questions about acute, right, obtuse, and reflex angles in polygons.

1. Can a quadrilateral have a reflex angle? Can all four of its angles be acute?
2. What are the possible combinations of angle types in a pentagon?
   Give reasons for your answers.

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Problem 2
Teacher's Guide

Topics

Polygons; classifying angles; interior angles in polygons

Materials

Rulers (for drawing sides of polygons); protractors (if students ask for them)

The grade level notes below are a rough guide for working with different age groups. Read through all three grades to get a feel for the likely range of students' thinking. Their approaches may vary a lot, so never feel limited to a particular grade level. Observe your students' ideas and strategies to assess their thinking, and build from there.

Grade 4 Common Core Standards: 4.MD.C 4.G.A.2

This problem gives students experience with problems that have no solutions and many solutions. If students are not familiar with acute, right, obtuse, straight, and reflex angles, they will need to research the definitions. (Reflex angles have measures between 180° and 360°.) It is possible to answer the questions without measuring angles as long as students can visualize their appearance and sketch them. For example, an acute angle is smaller than a right angle, and an obtuse angle is between a right angle and a straight angle.

Grade 5 Common Core Standards: 5.G.B

Students in fifth grade may be more likely to use protractors to measure angles to the nearest degree. They may wonder if there is a largest sum that will allow the polygon to close. By experimenting with different polygons, they may be able to predict that this sum depends on the number of sides. If they know that the angles in a triangle always have a sum of 180°, they may even predict what this sum is for quadrilaterals (360°), pentagons (540°), and other polygons.

Grade 6 Common Core Standards: ---

Students who are familiar with the patterns for sums of interior angles in polygons may be able to reason further about their conclusions. For example, a right, obtuse, and reflex angle already have a sum greater than 90° + 90° + 180° = 360°, which is the sum of the interior angles in a quadrilateral. Therefore, there is no "room" for the acute angle. On the other hand, the sum for pentagons is 540°, which allows for additional angles to be included.

Solutions begin on page 39.

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Name ______________________________

Problem 3

Complete puzzle #1 to find the *magic number* in the box. Explain why you think it is called a magic number. Then complete puzzles #2, #3, and #4.

#1	x ↓	÷ ↓	? ↓
÷ →	40	10	
x →	6	2	
? →			

#2	x ↓	÷ ↓	? ↓
÷ →			
x →			
? →			2

#3	x ↓	÷ ↓	? ↓
÷ →			
x →			
? →			100

#4	x ↓	÷ ↓	? ↓
÷ →			
x →			
? →			256

Observations and explanations:

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Name ______________________________

Problem 3: *Testing the Waters*

Complete puzzle #1 to find the *magic number* in the box. Explain why you think it is called a magic number. Then complete puzzles #2, #3, and #4.

#1	+ ↓	− ↓	? ↓
− →	40	10	
+ →	6	2	
? →			[ ]

#2	+ ↓	− ↓	? ↓
− →			
+ →			
? →			[2]

#3	+ ↓	− ↓	? ↓
− →			
+ →			
? →			[100]

#4	+ ↓	− ↓	? ↓
− →			
+ →			
? →			[256]

Observations and explanations:

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Name ______________________________

Problem 3: *Diving Deeper*

Choose one or more questions to answer, or think of your own new questions to explore.

1. Find more ways to get the magic numbers 48, 2, 100, and 256. Find patterns in your solutions.
2. Choose your own magic numbers and find solutions for them.
3. Does the puzzle still work if some of the numbers are decimals? Fractions?
4. Suppose the four upper-left corner numbers are *a*, *b*, *c*, and *d* (reading left to right, top to bottom). Find one or more expressions for the magic number. Do these expressions make it easier to find new solutions to puzzles #2, 3, and 4?

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Problem 3
Teacher's Guide

Topics

Properties of multiplication and division; patterns

Materials

None

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Grade 4 Common Core Standards: 4.NBT.A 4.NBT.B 4.OA.B.4 4.OA.C.5

After students finish the first puzzle and understand what magic numbers are, encourage them to spend plenty of time on puzzle #2, searching for multiple solutions and describing patterns between them. Students are likely to spend a fair amount of time guessing randomly at first. Teach them not to erase unsuccessful attempts on their thinking paper. When something doesn't work, suggest that they think about how to adjust it instead of starting over with four new numbers.

Grade 5 Common Core Standards: 5.OA.A 5.NBT.B.5 5.NBT.B.6

However, since students in grade 5 may be more fluent with the computations, they might complete more of the puzzles, even while thinking more systematically than younger students. Ask them to explain what happens to the magic number when they double, triple, take half, take a third, etc. of one or more of the numbers. (See the suggestions at the end of the Solutions.)

Grade 6 Common Core Standards: 6.NS.B.2 6.EE.A

Sixth grade students may be able to look more closely at number and operation properties and to express the patterns with variables. See the final Diving Deeper question and its solution on page 48.

Solutions begin on page 41.

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Name ______________________________

Problem 4

The quotient is 163, and the remainder is 7. The divisor is less than 10.
What are the dividend and divisor?

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Name ______________________________

Problem 4: *Testing the Waters*

The quotient is 13, and the remainder is 7. The divisor is less than 10.
What are the dividend and divisor?

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Name ________________________________

Problem 4: *Diving Deeper*

Choose one or more questions to answer, or think of your own new questions to explore.

1. Find more solutions for divisors greater than or equal to 10. What patterns do you see? How many solutions are there?
2. Make a table, formula, and graph for the divisor (input) and the dividend (output).
3. Do the same for other quotients and remainders. Compare the results.

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Problem 4
Teacher's Guide

Topics

Division; remainders; relationship between multiplication and division; patterns

Materials

None

The grade level notes below are a rough guide for working with different age groups. Read through all three grades to get a feel for the likely range of students' thinking. Their approaches may vary a lot, so never feel limited to a particular grade level. Observe your students' ideas and strategies to assess their thinking, and build from there.

Grade 4 Common Core Standards: 4.NBT.B.6

Strategy #1 in the Solutions shows a likely approach. Students may need time to realize that the divisor must be greater than 7.

Students who have not yet learned procedures for dividing by one-digit numbers can work on this problem, but much of their thinking will go toward developing division strategies, which changes the focus of the problem. This is fine—there is no need to teach "long division" in advance! Just know that they may not make as much progress on the rest of the problem.

Grade 5 Common Core Standards: 5.OA.A 5.NBT.B

Students in grade 5 may be more likely to use the relationship between multiplication and division in their thinking process. See Strategy #2 in the Solutions.

Grade 6 Common Core Standards: 6.NS.B.2 6.NS.C.8 6.EE.C.9

Encourage more advanced students to extend the problem by investigating divisors greater than or equal to 10 and looking for patterns. See the Diving Deeper questions and their solutions.

Solutions begin on page 43.

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Name ______________________________

Problem 5

Fill in the blank with the best place value name or number.

The number of base ten unit cubes that would fill my classoom is close to ___________________________.

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Name ______________________________

Problem 5: *Testing the Waters*

Fill in the blank with the best place value name or number.

The number of Base Ten unit cubes that would cover my classroom floor is close to _________________________.

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Name ________________________________

Problem 5: *Diving Deeper*

Choose one or more questions to answer, or think of your own new questions to explore.

About how many Base Ten unit blocks would it take to fill your

- Cafeteria?
- Gymnasium?
- School?

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Problem 5
Teacher's Guide

Topics

Place value; large numbers; multiplication and division

Materials

Base ten blocks (recommended)

Note: Each base ten unit cube (or "unit") is 1 cm by 1 cm by 1 cm (1 cubic centimeter). Ten units lined up in a row form a "long." Ten longs joined to make a square form a "flat." Ten flats joined to make a larger cube form a "big cube."

The grade level notes below are a rough guide for working with different age groups. Read through all three grades to get a feel for the likely range of students' thinking. Their approaches may vary a lot, so never feel limited to a particular grade level. Observe your students' ideas and strategies to assess their thinking, and build from there.

Grade 4 Common Core Standards: 4.NBT.A

Grade 4 students may need to have physical base ten blocks available in order to visualize a solution process. See Strategy #1 in the Solutions for an approach that does not require a lot of measurement or multiplication.

Grade 5 Common Core Standards: 5.NBT.A 5.NBT.B 5.MD.A 5.MD.C

See Strategy #2 in the Solutions for another likely approach. It may help to note that 1 meter is slightly longer than 1 yd.

Consider discussing different ways to name the metric measures of the actual blocks as well as the ones that they imagine. For example:

What is the volume of a big cube? (1000 cubic centimeters, 1 cubic decimeter, 0.001 cubic meters)
What is the volume of a "big flat*"? (100,000 cubic centimeters, 0.1 cubic meters)
What is the volume of a "big big cube"? (1 million cubic centimeters, 1 cubic meter, etc.)

Grade 6 Common Core Standards: 6.G.A.2

Older students may be more likely to attempt the Diving Deeper questions. Also, they may have less need to work with physical blocks, though they may still find it helpful at least to see an example of a "big cube."

Solutions begin on page 45.

* See Strategy #1 in the Solutions

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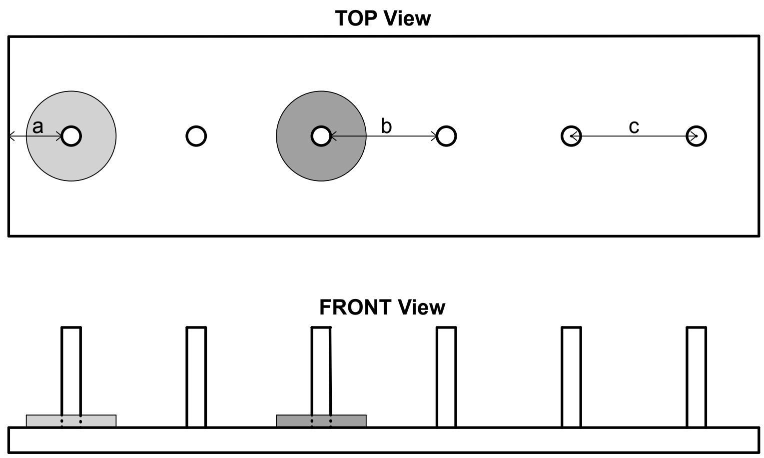
Name ______________________________

Problem 6

You are inventing a game in which players move disks between pegs. The game board is a 15-inch by 4-inch rectangle with six equally-spaced pegs. Each peg has a *diameter* of $\frac{3}{8}$ inches. The distance, a (on the left and right), is half the distance, b, between pegs.



What are the measurements, a and b?
What is the distance, c, between the *centers* of the pegs?

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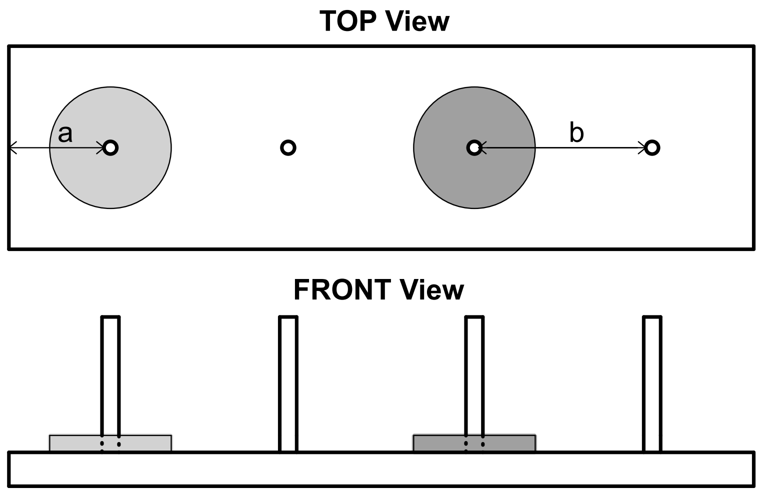
Name ______________________________

Problem 6: *Testing the Waters*

You are inventing a game in which players move disks between pegs. The game board is an 11-inch by 3-inch rectangle with four equally-spaced pegs. Each peg has a *diameter* of $\frac{1}{4}$ inches. The distance, a (on the left and right), is half the distance, b, between pegs.



What are the measurements, a and b?

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Problem 6: *Diving Deeper*

Choose one or more questions to answer, or think of your own new questions to explore.

1. How far are the pegs from the front and back sides of the game board?
2. What is the largest possible diameter of a disk so that it does not overlap other disks?
3. Invent a game using this game board. How many disks will each player have? Will all disks be the same size? The same color? What are the rules? How will a player win?

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Problem 6
Teacher's Guide

Topics

Equivalent fractions; measuring lengths; adding and subtracting fractions; finding fractions of numbers

Materials

Rulers; graph paper; Problem 6 Number Line Handout on page 59 (at least one of these three items)

The grade level notes below are a rough guide for working with different age groups. Read through all three grades to get a feel for the likely range of students' thinking. Their approaches may vary a lot, so never feel limited to a particular grade level. Observe your students' ideas and strategies to assess their thinking, and build from there.

Grade 4 Common Core Standards: 4.NF.A 4.NF.B 4.MD.A.2

Although many grade 4 students will not have learned to add and subtract fractions with unlike denominators, they can still work on the problem. In fact, the problem will help them to visualize the calculations on a number line and perhaps to discover some of their own procedures. If the numbers are too complex, have them try the Testing the Waters problem first or instead.

If the marks on the ruler are hard to read because they are so close together, have your students use the number lines on the Problem 6 Number Line Handout on page 59 or make a picture of two or three inches of a ruler on graph paper. They will need to figure out how to mark the scales on their lines.

Grade 5 Common Core Standards: 5.NF.A 5.NF.B.4a 5.NF.B.6 5.NF.B.7

Although fifth graders are more likely to know procedures for adding and subtracting fractions, they will still benefit from looking at rulers and number lines to *see* what the procedures mean and why they make sense. They may need to use repeated addition to find the total of the pegs' diameters, and they will probably need to use some combination of Strategies #1, #2, and #3 to find one half and one sixth of a fraction or mixed number. Some students may still find it helpful to begin with Testing the Waters.

Grade 6 Common Core Standards: ---

Sixth graders may be able to solve the problem more quickly. They will be more fluent with adding and subtracting fractions and they may even use procedures for multiplying and dividing fractions, but they will still benefit from using number lines to explain why their answers make sense.

Solutions begin on page 47.

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Name ______________________________

Problem 7

Describe a real-world situation to match the equation $(10 \bullet x + 5 \bullet y) \div 2 = z$.
Be sure to explain what x, y, and z stand for.

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Name ________________________________

Problem 7: *Testing the Waters*

Describe a real-world situation to match the equation $3 \bullet x + y = z$.
Be sure to explain what x, y, and z stand for.

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Name ______________________________

Problem 7: *Diving Deeper*

Choose one or more questions to answer, or think of your own new questions to explore.

1. Create at least one new expression with *x* and *y* that always gives the same answer for z as the original one. Test your expression(s).
2. Explain *why* your expression(s) always gives the same result.

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Problem 7
Teacher's Guide

Topics

Interpreting algebraic expressions; properties of operations; order of operations

Materials

None

The grade level notes below are a rough guide for working with different age groups. Read through all three grades to get a feel for the likely range of students' thinking. Their approaches may vary a lot, so never feel limited to a particular grade level. Observe your students' ideas and strategies to assess their thinking, and build from there.

Grade 4 Common Core Standards: ---

Grade 4 students may not know that multiplication is traditionally done before addition. Instruct these students to insert parentheses:

$$((10 \bullet x) + (5 \bullet y)) \div 2 = z$$

As students create their stories, encourage them first to find the four steps in the calculation and to think about what happens first, what happens next, etc. The Testing the Waters question has a simpler expression in case this one is too complex.

Grade 5 Common Core Standards: 5.OA.A

If students work on the Diving Deeper question, encourage them to think about how the expression might change if they did the steps in a different order. Would any of the numbers change? Would the parentheses change? Emphasize that their expression(s) must *always* give the same answer as the original, no matter what numbers they choose for x and y. Expressions that do this are called *equivalent expressions*.

Grade 6 Common Core Standards: 6.EE.A

Students who are familiar with leaving out the multiplication symbol may rewrite the expression as

$$(10x + 5y) \div 2 = z.$$

Some of them may also express the division as a fraction.

Older students may be more likely to include fractions or decimals in their expression as they try the Diving Deeper tasks. They may be more purposeful about using known properties (commutative, associative, and perhaps distributive) to generate equivalent expressions. Encourage them to use the names of these properties in their explanations.

Solutions begin on page 49.

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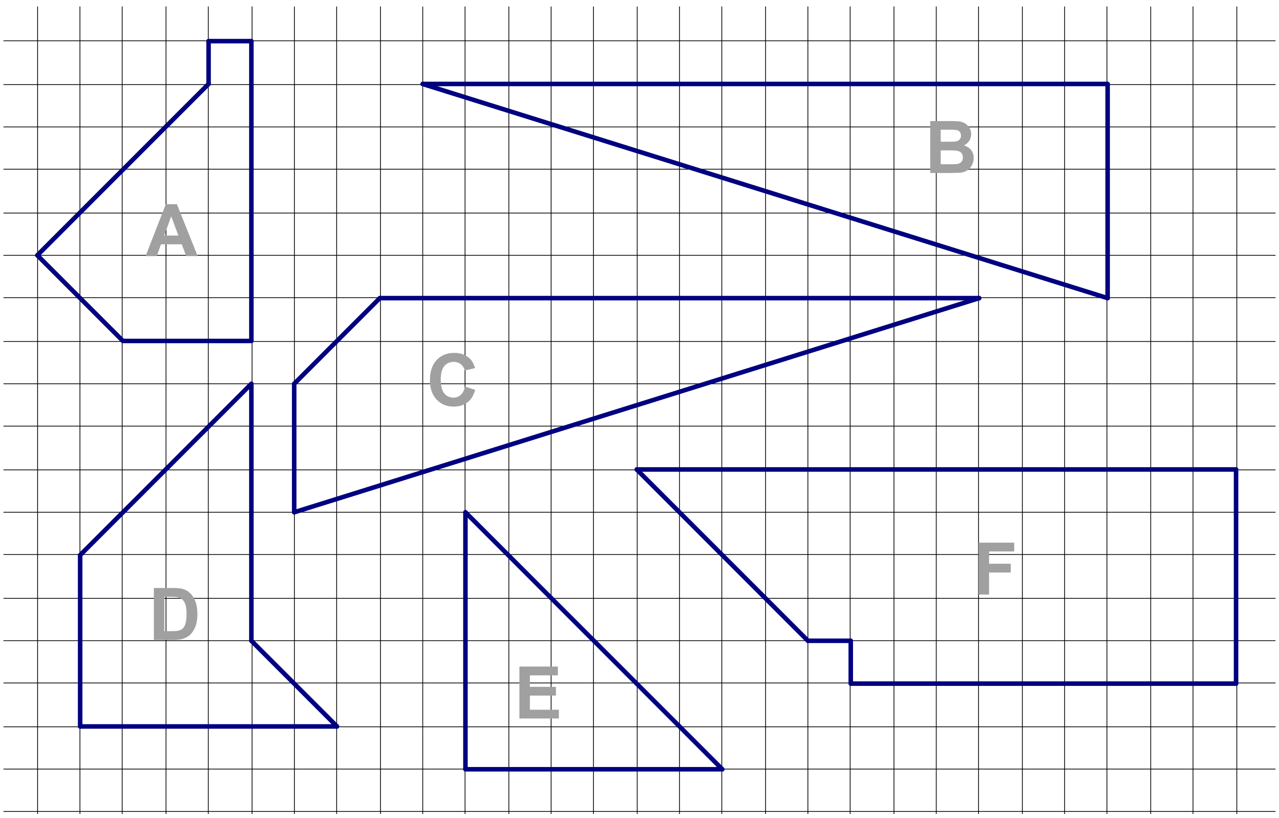
Name ________________________________

Problem 8

All of these pieces combine to make ONE whole. What fraction and decimal is each piece?
Hint: The pieces can be arranged to make a rectangle.



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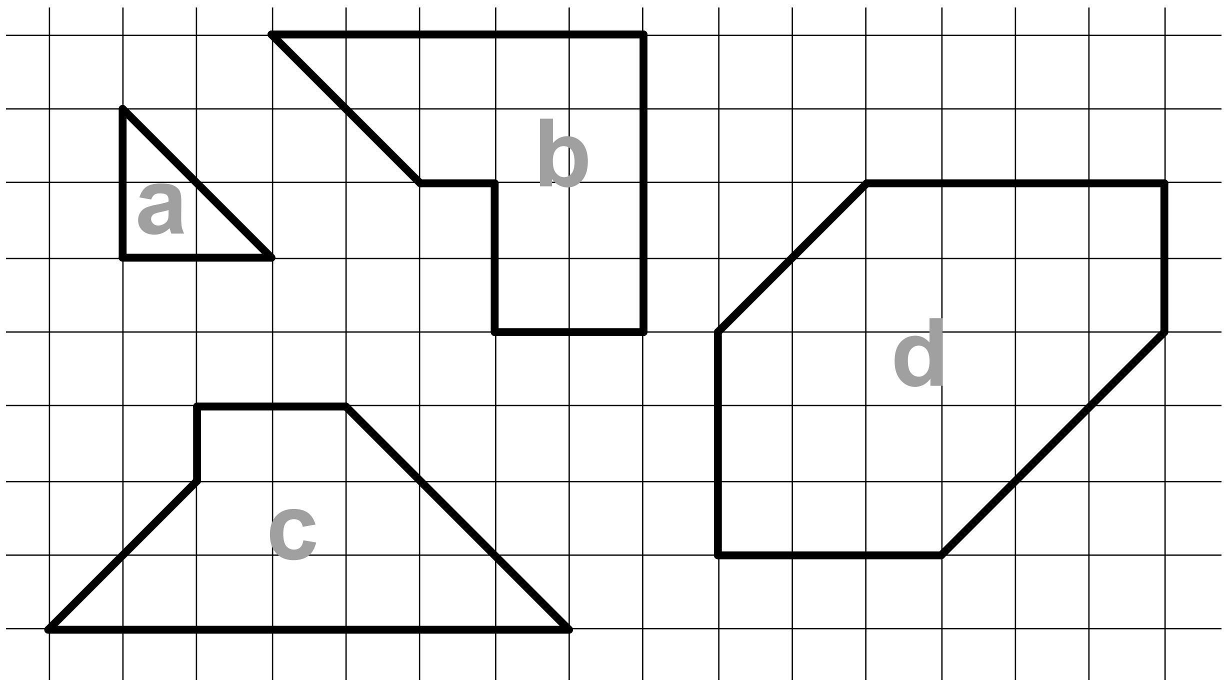
Name ______________________________

Problem 8: *Testing the Waters*

All of these pieces combine to make ONE whole. What fraction and decimal is each piece?
Hint: The pieces can be arranged to make a rectangle.



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Name ________________________________

Problem 8: *Diving Deeper*

Choose one or more questions to answer, or think of your own new questions to explore.

Create your own challenging puzzles like this. Trade puzzles with other students and solve them. Suggestions:

1. Make the denominator more challenging by changing the area of the main rectangle.
2. Use shapes having fewer horizontal, vertical, and “45° diagonal” sides.
3. What kinds of areas for the main rectangle will ensure that equivalent fractions are the most helpful strategy for finding the decimals? Why? What do you notice about the decimals for these fractions?

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Problem 8
Teacher's Guide

Topics

Equivalent fractions; area; writing fractions as decimals; geometric transformations; spatial visualization

Materials

A copy of the Recording Page (page 60) for each student; scissors (for students who want to cut the shapes out to rearrange them); graph paper (for students who want to draw the shapes as they work)

Tip: Students may find it easier to cut out and rearrange the shapes if you copy Problem 8 onto card stock.

The grade level notes below are a rough guide for working with different age groups. Read through all three grades to get a feel for the likely range of students' thinking. Their approaches may vary a lot, so never feel limited to a particular grade level. Observe your students' ideas and strategies to assess their thinking, and build from there.

Grade 4 Common Core Standards: 4.NBT.A.1 4.NF.A 4.NF.C 4.MD.A.3

Younger students may find it especially helpful to cut the shapes out and rearrange them. Some of the shapes contain built-in clues (parts that seem to fit together naturally).

Shapes A, D, E, and F are made entirely from whole- and half-squares, making it easier to count the unit squares inside them. Some students may use the rectangle area formula to reduce the amount of counting they have to do for some parts of the shapes.

Shapes B and C are more challenging, because the long diagonal sides cut the unit squares into awkward-looking pieces. Fourth graders may be less likely to find fractions and decimals for this part of the puzzle. See the grade 5 section for students who are ready to take this step.

Grade 5 Common Core Standards: 5.NBT.A 5.NBT.B 5.NF.A

This is a great problem for students to work on just *before* they learn a formula for the area of a triangle. By imagining the rectangle that B and C *almost* make when you join them (there is a small right triangle missing from one corner), students can *see* that the area of Triangle B is half the area of a rectangle. There is no need for them to decompose the triangle into awkward bits of unit squares!

Grade 6 Common Core Standards: 6.G.A.1

Some older students may not need to cut the pieces out in order find a way to make a rectangle from them. They may also use area formulas for rectangles and triangles to make their work more efficient. Many grade 6 students may work on the Diving Deeper questions.

Solutions begin on page 51.

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Intrepid Math Solutions

Problem 1

Part 1

Staci chose a number in the form 9*a*1. (*a* represents any digit.)
9*a*1 means 901, 911, 921, 931, 941, 951, 961, 971, 981, or 991.

Numbers in the form 8*a*0 are also solutions if you accept the idea that 0*a*8 means *a*8!

Part 2

Staci chose a number that reads the same forward and backward (a *palindrome*). In other words, she chose a number whose ones and hundreds digits are the same.

Examples: 646 or 737.

Part 3

There are many solutions: 5*a1*, 6*a*2, 7*a*3, 8*a*4, and 9*a*5.
Some students may notice that the hundreds digit minus the ones digit is always 4.

Part 4

0, 99, 198, 297, 396, 495, 594, 693, 792, 891
The possible answers are the first ten multiples of 99 (including 0).

Notes

There are all sorts of nice patterns in the digits of these numbers. For example:

- The tens digit is always 9 (except for 0).
- The ones and hundreds digits have a sum of 9 (except for 0).
- The ones digits decrease by 1 each time (starting with 99).
- The hundreds digits increase by 1 each time (starting with 198).
- If you remove the tens digit from each numeral, you get multiples of 9.

Some students may notice that they can predict which multiple of 99 they will get by multiplying 99 by the difference between the first and last digits!

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Problem 1: *Testing the Waters*

Part 1

Staci chose the number 91 (or 80 if you think of ‘08’ as 8).

Part 2

Staci chose a number whose digits are the same. Examples: 55, 77

Part 3

There are many solutions: 51, 62, 73, 84, and 95.

Part 4

0, 9, 18, 27, 36, 45, 54, 63, 72, 81 (multiples of 9)

Problem 1: *Diving Deeper*

1. The problem limits Staci’s choices to numbers greater than 500 in order to prevent duplicate solutions. For example, 297 and 792 form a solution pair. One of the numbers is greater than 500, and the other is less than 500.

2. If you solve the problem for four-digit numbers whose two middle digits are equal, you will get multiples of 999.

3 and 4.

Situations in which the two middle digits are not equal and numbers with five or more digits are more complicated. There are many things to discover. I will leave this to you!

5. Some students may be able to give general reasons that the answers for the original problem always multiples of 99. Those who have learned algebraic procedures may be able to prove it. If the hundreds digit is n greater than the ones digit, then

$$(100a + 10b + a - n) - (100(a - n) + 10b + a) =$$
$$(100a + 10b + a - n) - (100a - 100n + 10b + a) =$$
$$(101a + 10b - n) - (101a - 100n + 10b) =$$
$$101a - 101a + 10b - 10b - n + 100n =$$
$$99n.$$

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Problem 2

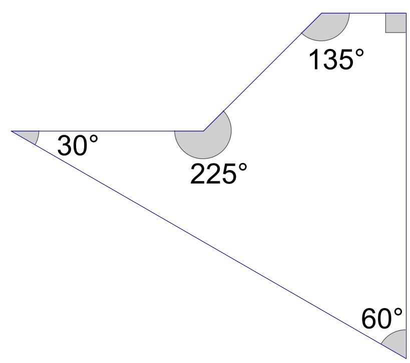
Part 1

There is no solution. Since there are exactly four angles, the polygon has to be a quadrilateral. The angles are too large for it to close with only four sides.

Some students may wonder if the polygon could contain a straight angle. They will probably agree that this does not make sense since the two sides of the angle would look like one side of the polygon.

Part 2

Sample solution



Since there can be more than one of each type of angle, the polygon can have more sides. This makes it possible to close the polygon. (Students' solutions may include polygons with more than five sides.)

Notes

You may or may not require students to show the measures of their angles.

The Common Core focuses strongly on concepts around angle measurement in grade 4, but does not have much to say about them in grades 5 and 6. However, students' understandings in this area can continue to develop. For example, some students may be learning in their curriculum about sums of interior angles in polygons. In particular, this sum is 180° for triangles, and it increases by 180° each time you add another side. This problem provides opportunities for students to explore these ideas in various ways depending on their backgrounds.

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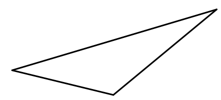
Problem 2: *Testing the Waters*

Part 1

No solution. The angles are too big. The sides would not "close up."

Part 2

Sample solution:



Part 3

No solution. When you join the two sides of the angle to make a triangle, the reflex angle will always be outside of it.

Students who know that the angles inside a triangle always have a sum of 180° may use this knowledge in their explanations.

Problem 2: *Diving Deeper*

1. Yes, a quadrilateral can have a reflex angle.
   No, its four angles cannot all be acute.

2. There are many possible combinations of angle types in a pentagon. I leave it to you to find as many as you can and to think about how you can tell when you have found them all.

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Problem 3

Sample solutions:

#1

	x	÷	x
÷	40	10	4
x	6	2	12
÷	240	5	48

#2

	x	÷	x
÷	6	6	1
x	2	1	2
÷	12	6	2

#3

	x	÷	x
÷	60	12	5
x	5	4	20
÷	300	3	100

#4

	x	÷	x
÷	320	160	2
x	4	32	128
÷	1280	5	256

There are many solutions for #2, #3, and #4.

The number in the box is called a magic number because when you multiply the numbers in the right column, and you divide the numbers on the bottom row, you always get the same answer!

Suggestions

Have students share their solutions to puzzles #2, 3, and 4. Ask new questions such as:
How many solutions does each puzzle have?
What do solutions to the same puzzle have in common?
What happens to the magic number if you double the upper left number?
What happens to the magic number if you double the upper middle number?
What happens to the magic number if you double all four of the upper left numbers?
What causes these changes?
Are the answers to these questions the same for all four puzzles?
Does anything predictable happen when you *add 1* to these numbers?

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Problem 3: *Testing the Waters*

Sample solution:

#1	+ ↓	− ↓	+ ↓
− →	40	10	30
+ →	6	2	8
− →	46	8	38

#2	+ ↓	− ↓	+ ↓
− →	8	7	1
+ →	0	1	1
− →	8	6	2

#3	+ ↓	− ↓	+ ↓
− →	110	70	40
+ →	40	20	60
− →	150	50	100

#4	+ ↓	− ↓	+ ↓
− →	300	244	56
+ →	56	144	200
− →	356	100	256

There are many solutions for #2, #3, and #4.
Students who continue on to the original problem should notice that the multiplication / division patterns are almost identical to the addition / subtraction patterns!

Problem 3: *Diving Deeper*

1. One way to get more solutions for the magic numbers 48, 2, 100, and 256 is to change one number in your original solution and then adjust another number (or numbers) until it gives the same magic number.
2. To make the problem more interesting, choose magic numbers that have some connection to numbers you have already solved for. For example, make the magic number 1 greater, 10 times greater, etc. Find predictable ways to adjust the numbers in your solutions.
3. Yes, the puzzle should still work if some of the numbers are decimals or fractions.
4. Students may create expressions such as

$(a \div b) \bullet (c \bullet d)$ $\qquad$ $(a \bullet c) \div (b \div d)$ $\qquad$ $a \bullet c \bullet d \div b$.

All of these expressions are *equivalent*. (They give the same answer when you use the same values for a, b, c, and d.) Some students may experiment with using parentheses in different ways. If they use fractions to stand for division, they may see connections to rules for multiplying and dividing fractions.

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Problem 4

There are two solutions.

(1) Divisor: 8 Dividend: 1311 Expression: 1311 ÷ 8

(2) Divisor: 9 Dividend: 1474 Expression: 1474 ÷ 9

8 and 9 are the only possible divisors. Numbers less than or equal to 7 cannot be divisors when the remainder is 7.

Strategies

#1: Estimate, test, and revise.

Example: Choose a divisor of 8. Estimate a dividend of 2000.

2000 ÷ 8 = 250 (too large)

1000 ÷ 8 = 125 (too small)

163 is closer to 125 than to 250, so the dividend should be closer to 1000 than 2000.

Continue adjusting until you reach a quotient of 163, then make small adjustments to get the remainder of 7.

#2: Use the inverse relationship between division and multiplication.

Choose a divisor, multiply it by the quotient. Add the remainder to get the dividend.

Example: Choose 8.

$$8 \bullet 163 + 7 = 1311$$

Some students may discover this strategy after first estimating and testing as in Strategy 1.

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Problem 4: *Testing the Waters*

There are two solutions.

(1) Divisor: 8 Dividend: 111 Expression: 111 ÷ 8
(2) Divisor: 9 Dividend: 124 Expression: 124 ÷ 9

8 and 9 are the only possible divisors. Numbers less than or equal to 7 cannot be divisors when the remainder is 7.

Problem 4: *Diving Deeper*

Divisor	Dividend	Expression
8	1311	1311 ÷ 8
9	1474	1474 ÷ 9
10	1637	1637 ÷ 10
11	1800	1800 ÷ 11

Whenever the divisor increases by 1, the dividend increases by 163.

Students who have studied the concept of *slope* and who make tables, formulas, and graphs may recognize that the *rate of change* of the relationship (the *slope* of the graph) is 163.

Students who have studied *y-intercepts* may recognize that the y-intercept of the graph is 7 (or would be if the graph extended that far).

By exploring further, students can discover that these connections between the quotient and the slope on one hand and between the remainder and the y-intercept on the other continue to hold true for other choices of quotient and remainder.

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Problem 5

Most classrooms will hold a number of unit cubes in the (low) hundred-millions.

For example, a room with a 10-meter by 5-meter rectangular floor and a 3-meter height has volume of 150 cubic meters. Each cubic meter contains 1 million unit cubes. Therefore, the classroom would hold 150 million cubes.

Suggestions

If you do not have base ten blocks, ask teachers from grades 1–3 if they have some that you can borrow. Otherwise, tell your students that a unit cube is 1 centimeter on each edge. Imagine and discuss the sizes and measurements of the other blocks.

Stress the importance of clear explanations in students' write-ups.

Strategies

#1: Visualize.
Students may not need to do much calculating. Start by picturing the "big cube" with 1000 unit cubes. Imagine placing 10 of these in a row in order to make a "big long" with 10,000 units. Then picture putting 10 big longs together to make a "big flat" containing 100,000 units. Visualize stacking 10 big flats into order to make a "big big cube," etc. All along the way, try to picture how much room each structure takes up, and continue until you have imagined filling up the room.

#2: Estimate the dimensions of the room.
Some students may also estimate the length, width, and height of the room and multiply these estimates (using mental math as much as possible). A practical approach may be to make the estimates in meters and then multiply each dimension by 100 to convert it to centimeters. However, students should initially make their own decisions about units.

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Problem 5: *Testing the Waters*

The number of unit cubes needed to cover most classroom floors is in the hundred-thousands.

Problem 5: *Diving Deeper*

Answers may vary.

Rather than starting over, students could begin by estimating the size of their classroom compared to the sizes of the cafeteria, gymnasium, or school.

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Problem 6

Length a: $1\frac{1}{16}$ inches
Length b: $2\frac{1}{8}$ inches
Length c: $2\frac{1}{2}$ inches

A helpful observation

You can combine the two halves on the left and right sides of the game board to make one whole. Join this whole with the five whole gaps between the pegs to make six “gaps.”

The general approach

Length of b: 3/8 • 6 = 2¼ 15 – 2¼ = 12¾ 12¾ ÷ 6 = 2 1/8
Length of a: half of 2 1/8 is 1 1/16
Length of c: 2 1/8 + half of 3/8 + half of 3/8 = 2 1/8 + 3/8 =2½

Fraction computation strategies

#1: Visual strategies
Look at a ruler or a number line, or make pictures of these on graph paper. See page 19 for some prepared number lines that students can label themselves.
For 3/8 • 6: Put together 6 groups of 3/8 and notice that you land at 2 1/4.
For 15 – 2 1/4 : Start at 15 and move left 2 and then another 1/4, landing at 12 1/4.

#2: Strategies based on thinking of denominators as units
For 3/8 • 6: Think of 3/8 as 3 eighths. Just as 6 groups of 3 people makes 18 people, 6 groups of 3 eighths makes 18 eighths. 18 eighths are 8 eighths + 8 eighths + 2 eighths; 1 + 1 + 1/4, or 2 1/4 .

#3: Strategies based on decomposition
For 2 1/8 ÷ 2: Decompose 2 1/8 into 2 + 1/8, and take half of each part. Half of 2 is 1, and half of 1/8 is 1/16. 1 + 1/16 = 1 1/16
For 12 3/4 ÷ 6: Decompose 12 3/4 into 12 + 3/4 , and divide each part by 6. 12 ÷ 6 = 2. You may deal with 3/4 ÷ 6 using Strategy #1 or #2 to get 1/8. 2 + 1/8 = 2 1/8.

For ¾ ÷ 6: Using Strategy #1, it is easy to see how to split ¾ into six equal parts on the number line on page 19. Each part is 1/8. Using Strategy #2, think of 3/4 as 6/8 or 6 eighths. One sixth of 6 eighths is 1 eighth.

#4: Traditional strategies:
Use the traditional rules for computing with fractions.

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Problem 6: *Testing the Waters*

Length of a : $1\frac{1}{4}$ inches
Length of b: $2\frac{1}{2}$ inches

Problem 6: *Diving Deeper*

1. The pegs are 1 13/16 inches from the front and back sides of the game board.

2. The largest possible diameter of a disk so that it does not overlap other disks is $2\frac{1}{2}$ inches. Notice that this is the same as the distance, c, between the centers of neighboring pegs.

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Problem 7

A sample story

Shawna's family saves their extra dimes and nickels. At the end of each year, they share what they have saved equally between two charities. x is the number of dimes they save; y is the number of nickels they save; and z is the amount of money in cents that each charity receives.

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Problem 7: *Testing the Waters*

A sample story

A specialty store sells tricycles and unicycles. If *x* equals the number of tricycles and *y* equals the number of unicycles in the store, then *z* equals the number of wheels in the store (assuming there are no other types of vehicles with wheels)!

Problem 7: *Diving Deeper*

Some equivalent expressions

(5 • *x*) + (5 • *y* ÷ 2)	(2 • *x* + *y*) • 5 ÷ 2
((2 • *x* + *y*) • 5) ÷ 2	(2 • *x* + *y*) • 2.5

The parentheses in the upper-left expression are optional.

Examples of testing the expressions

Choose any number of dimes and nickels. For example: *x* = 200 and y = 40

Original expression
(10 • *x* + 5 • *y*) ÷ 2 =
((10 • 200 + 5 • 40)) ÷ 2 =
(2000 + 200) ÷ 2 =
2200 ÷ 2 =
1100

Upper-left expression
(5 • *x*) + (5 • *y* ÷ 2) =
(5 • 200) + (5 • 40 ÷ 2) =
1000 + 200 ÷ 2 =
1000 + 100 =
1100

Upper-right expression
(2 • *x* + *y*) • 5 ÷ 2 =
(2 • 200 + 40) • 5 ÷ 2
(400 + 40) • 5 ÷ 2 =
440 • 5 ÷ 2 =
2200 ÷ 2 =
1100

Lower-right expression
(2 • *x* + *y*) • 2.5 =
(2 • 200 + 40) • 2.5 =
(400 + 40) • 2.5 =
440 • 2.5 =
1100

Students who have not learned rules for multiplying decimals can think of 440 • 2.5 as two and a half groups of 440.

Two groups of 440 is 880.
Half of a group of 440 is 220.

Two and a half groups of 440 is 880 + 220 = 1100

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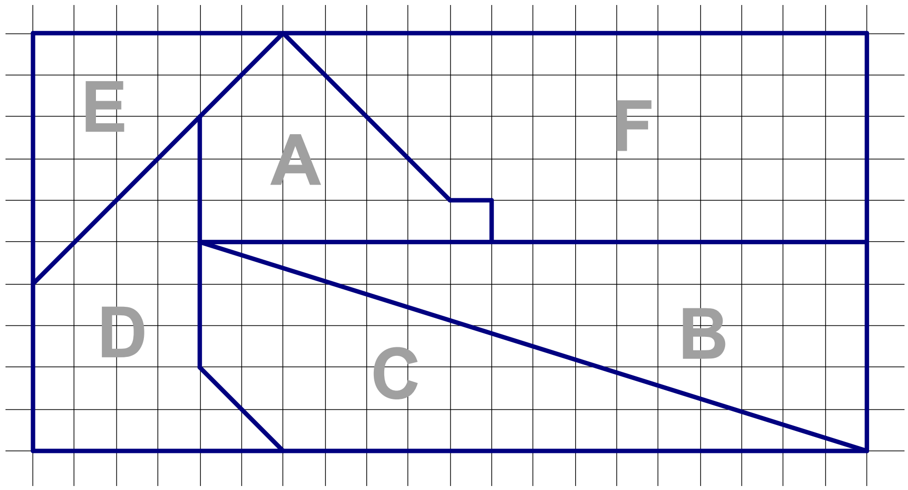
Problem 8



A $\frac{21}{200}$, 0.105 C $\frac{19}{100}$, 0.19 E $\frac{9}{100}$, 0.09

B $\frac{1}{5}$, 0.2 D $\frac{13}{100}$, 0.13 F $\frac{57}{200}$, 0.285

A typical process

(1) Put the pieces together to make a rectangle.
(2) Multiply 10 • 20 to find that the rectangle contains 200 unit squares.
(3) Use various strategies to count the unit squares in (find the area of) each shape.
(4) Form a fraction with a denominator of 200 for each shape. (Some students may simplify their fractions.)
(5) Write each fraction in decimal form.

Suggestions

Ask students if they can think of a way to check their answers for the fractions and decimals. (For example, verify that they have a sum of 1.)
Ask students to share, compare, and justify strategies for finding areas, fractions, and decimals.
Ask students to use equivalent fractions and/or the picture to help them find and justify their answers for the decimals, especially for shapes A and F. Example with shape A:
The fraction may be rewritten as $\frac{105}{1000}$ or $\frac{10.5}{100}$. How does each fraction lead to 0.105?
Since there are 200 unit squares, each one stands for 5 thousandths. To find the total number of thousandths, count each square in A as 5 and each half-square as 2.5.

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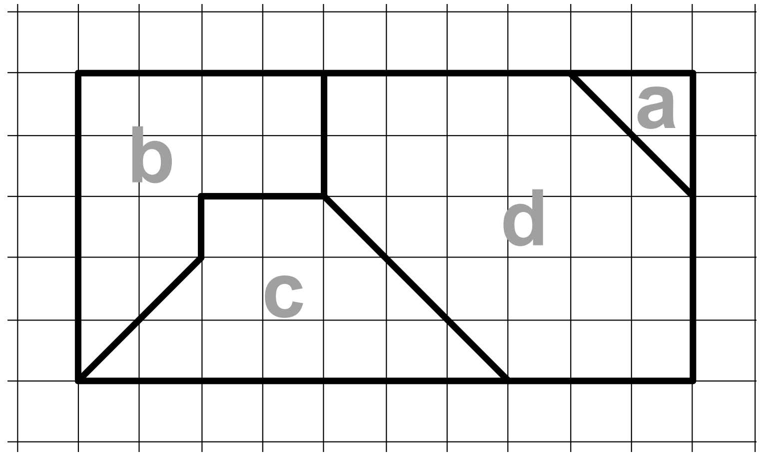
Problem 8: *Testing the Waters*



a $\frac{1}{25}$, 0.04 c $\frac{1}{4}$, 0.25

b $\frac{6}{25}$, 0.24 d $\frac{47}{100}$, 0.47

Problem 8: *Diving Deeper*

3. Rectangles whose areas have prime factors of only 2 and/or 5 will give denominators that can be multiplied by a whole number to create a power of ten (10, 100, 1000, 10000, etc.). This will result in a decimal that *terminates* (stops). Other whole number areas will result in repeating decimals.

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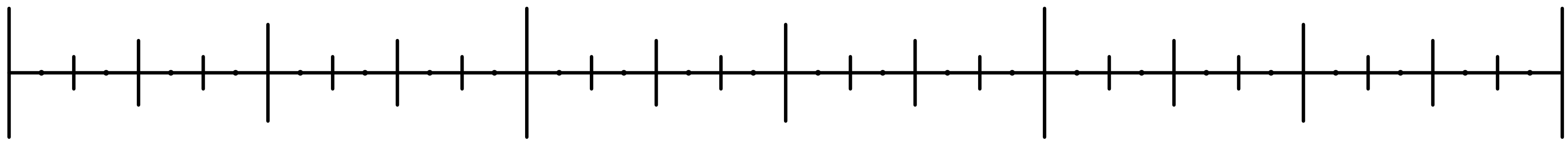
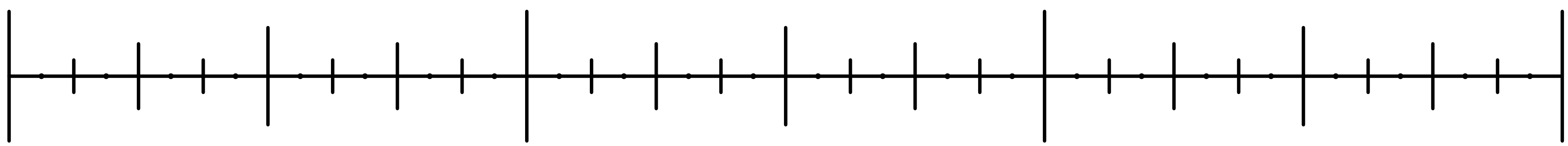
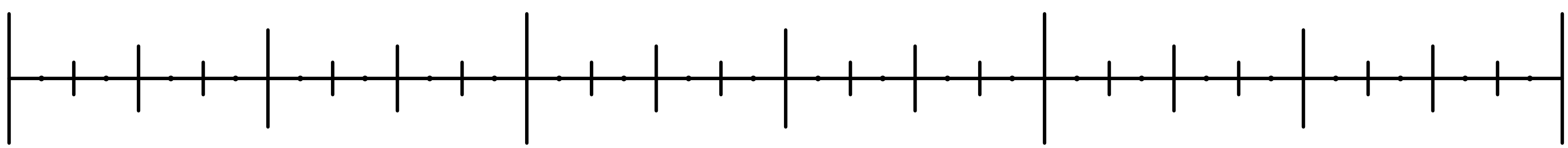
Problem 6
Number Line Handout







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Problem 8
Recording Page

Show how to arrange the pieces to make a rectangle.

Fractions, decimals, and explanations

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