Ten Plus One:

Strategies for Enhancing Depth and Complexity of Math Tasks

examples organized by strategy

by Jerry Burkhart

Strategy 1: Write a story.

One of the best ways to be certain that students understand the meaning of an operation is to have them write a story or a problem about it. In early grades, students may be doing this as part of their regular work. If so, you may enhance the complexity of the task (see the "New Task +") by using larger numbers, more numbers, more operations, etc. In later grades, writing a story about a concept or calculation is an exceptionally powerful strategy that is often overlooked.

If students are studying a concept or a definition, you may ask them to describe a related real-world situation or connection instead of a story.

Grade 1

Original task: 36 + 7
New Task: Write and answer a story problem for 36 + 7.
New Task +: Write and answer a story problem for 36 + 27.

Grade 2

Original task: 35 – 18
New Task: Write and answer a story problem for 35 – 18.
New Task +: Write and answer a story problem for 35 – 18 + 11.

Grade 3

Original task: 10 x 37
New Task: Write and answer a real-world story problem for 10 x 37.
New Task +: Write and answer a real-world story problem for 10 x 10 x 37.

Grade 4

Original task: 352 ÷ 6
New Task: Write and answer a real-world story problem for 362 ÷ 6.
New Task +: Write and answer a real-world story problem for 362 ÷ (6 x 2).

Strategy 1: Write a story.

Grade 5

Original task: Write >, <, or =.   0.7 ___ 0.58

New Task: Write a real-world story about comparing 0.7 and 0.58.

New Task +: Write a real-world story about comparing 0.07, 0.058, and 0.12.

Grade 6

Original task: $6 \div \frac{2}{3}$

New Task: Write and answer a real-world story problem for $6 \div \frac{2}{3}$.

New Task +: Write and answer a real-world story problem for $5 \div \frac{2}{3}$.

Grade 7

Original task: 13 – -5

New Task: Write and answer a real-world story problem for 13 – -5.

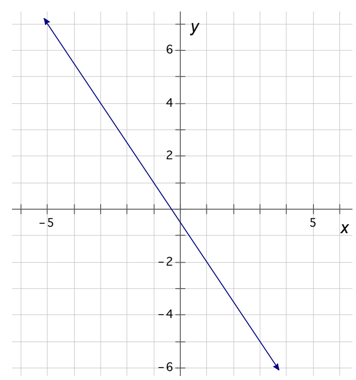
New Task +: Write and answer two real-world story problems for 13 – -5.

Use a different meaning of subtraction and a different real-world situation for each story.

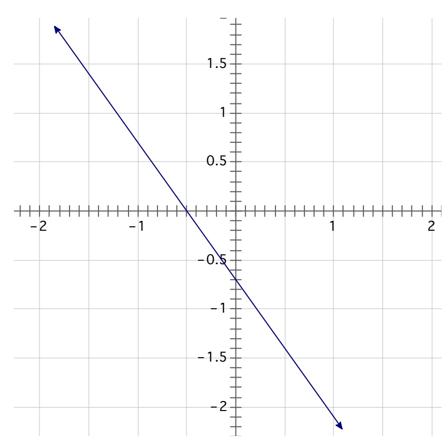
Grade 8

Original task: Find the slope of the line.

New Task: Write a real-world story about The line and its slope. Tell what *x, y,* and the slope represent.



New Task +: Write a real-world story about the line and its slope. Tell what *x*, *y*, and the slope represent.



Strategy 2: Draw a picture.

The purpose of drawing a picture is to illustrate mathematical meaning. You may prefer to say drawing a *diagram*, especially for older students—the idea being that a diagram is a drawing that shows mathematical ideas clearly and simply (without necessarily looking like a real-world object).

Grade 1

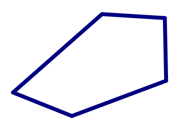
Original task: True or False? 7 + 4 = 6 + 5

New Task: Draw a picture or diagram to show that 7 + 4 is equal to 6 + 5.

New Task +: Draw a picture or diagram to show that 7 + 4 + 9 is equal to 9 + 6 + 5.

Grade 2

Original task: Name the shape.



New Task: Draw a pentagon whose sides are all different lengths.

New Task +: Draw five pentagons, and make each one as different as you can.

Asking students to make each pentagon as different as possible stretches them to test the limits of what it means for something to be a pentagon.

Grade 3

Original task: Fill in the box to make the fractions equivalent. $\frac{2}{3} = \frac{6}{\square}$

New Task: Draw a diagram to show that $\frac{2}{3}$ is equivalent to $\frac{6}{9}$.

New Task +: Draw a diagram to show that $\frac{6}{8}$ is equivalent to $\frac{9}{12}$.

(The New Task + is more complex because neither fraction is in simplest form.)

Grade 4

Original task: 62 ÷ 6

New Task: Draw a diagram showing the meaning of 62 ÷ 6 and the answer.

New Task +: Draw diagrams showing two meanings of 62 ÷ 6 and the answer.

Division expressions always have two meanings based on the idea of groups. In this case, the meanings are "If you divide 62 into 6 equal groups, how many are in each group?" and "How groups of 6 are in 62?" You can also use division to compare two numbers: "62 is how many times as much as 6?" In these cases, the answer is not a whole number, so students will have to account for the remainder.

Strategy 2: Draw a picture.

Grade 5

Original task: $6 \cdot \frac{2}{3}$

New Task: Draw a diagram showing the meaning and the value of $6 \cdot \frac{2}{3}$.

New Task +: Draw diagrams showing two meanings and the value of $6 \cdot \frac{2}{3}$ (or $5 \cdot \frac{2}{3}$).

Two important meanings are “6 groups of 2/3” and “2/3 of a group of 6.”

Grade 6

Original task: $6 \div \frac{2}{3}$

New Task: Draw a diagram showing the meaning and the value of $6 \div \frac{2}{3}$.

New Task +: Draw diagrams showing two meanings and the value of $6 \div \frac{2}{3}$ (or $6 \div \frac{4}{5}$).

Two important meanings are “number of groups of 2/3 in 6” and “the size of one (whole) group is 6 is 2/3 of a group. Some students may think of the inverse of multiplication: “What must I multiply 2/3 by to get 6?” or “2/3 or what is 6?”

Grade 7

Original task: 13 – -5

New Task: Draw a diagram showing the meaning and the value of 13 – -5.

New Task +: Draw two models that show different ways to understand the meaning of 13 – -5 and how to find its value.

Students usually show a number line diagram (indicating how far and in which direction to travel) or the “take away” meaning. They may also show a comparison (“how much more?”) meaning. There are many ways to think about subtraction with negative numbers!

Grade 8

Original task: Find the length of the hypotenuse of a right triangle with legs of 4 and 10 units.

New Task: On graph paper, draw a right triangle with legs of 4 units and 10 units. Draw a square on the hypotenuse and use it to find the length of the hypotenuse.

New Task +: On graph paper, draw a right triangle with legs of 4 units and 10.5 units. Draw a square on the hypotenuse and use it to find the length of the hypotenuse.

The New Tasks work well even if (or especially if?) students have not yet been taught the Pythagorean theorem. If you give the task to them before teaching the theorem, then you are also using Strategy 7: Solve to Learn.

Strategy 3: Explain why.

"Explain Why" is a sure-fire way to increase the depth of almost any math task. Because of this, it may be the most flexible strategy of all. It tends to combine well with all of the other strategies.

Grade 1

Original task: True or False: 7 + 4 = 6 + 5
New Task: Explain why 7 + 4 = 6 + 5 without finding the sums.
New Task +: Explain why 7 + 4 + 9 is equal to 9 + 6 + 5 without find the sums.

Grade 2

Original task: Write <, >, or =.   302 ____ 297
New Task: Write <, >, or =. 302____ 297. Explain why your answer makes sense.
New Task +: Explain why 302 – 14 < 297 – 8.

Grade 3

Original task: 10 x 37
New Task: Explain why the product of 10 x 37 looks like 37 followed by 0.
New Task +: Explain why the product of 100 x 37 looks like 37 followed by two 0s.

Grade 4

Original task: $\frac{5}{8}-\frac{1}{4}$
New Task: Explain why $\frac{5}{8}-\frac{1}{4}\neq\frac{5-1}{8-4}$.
New Task +: Explain why $\frac{7}{12}-\frac{4}{8}\neq\frac{7-4}{12-8}$.

Sometimes, explaining why *not* offers a more approachable way to get at a deep concept.

Strategy 3: Explain why.

Grade 5

Original task: $6 \cdot \frac{2}{3}$

New Task: Explain why $6 \cdot \frac{2}{3} = 4$.

New Task +: Explain why $6 \cdot \frac{3}{4} = 4\frac{1}{2}$.

Grade 6

Original task: $6 \div \frac{2}{3}$

New Task: Explain why $6 \div \frac{2}{3} = 9$.

New Task +: Explain why $6 \div \frac{4}{5} = 7\frac{1}{2}$.

Grade 7

Original task: 13 – -5

New Task: Explain why 13 – -5 > 13.

New Task +: Explain why -13 – -5 – 6 is one less than -13.

Grade 8

Original task: Decide if each number is rational or irrational:

$$\frac{3}{4} \quad \sqrt{10} \quad \frac{2.3}{0.8} \quad \pi \quad 0 \quad -6 \quad 2 \cdot \sqrt{9}$$

New Task: Explain why $\frac{2.3}{0.8}$ is a rational number.

New Task +: Explain why $\frac{2.3 - \frac{3}{4}}{0.8}$ is a rational number.

Strategy 4: Find another way.

Asking students to find another way to answer a question or solve a problem is an easy and effective way to help them to learn to connect mathematical ideas and become more flexible thinkers. They may resist at first, especially if they believe that answers are the most important thing in math, but it is important to change this mindset. After students have found more than one approach, you may use Strategy 5 and ask them to compare and contrast the solution methods. How are they the same or different? What are some advantages and disadvantages of each approach? Under what circumstances would one approach be better than the other?

Grade 1

Original task: 36 + 7
New Task: Find another way to calculate 36 + 7.
New Task +: Find three ways to calculate 36 +27.

Grade 2

Original task: 35 – 18
New Task: Find another way to calculate 35 – 18.
New Task +: Find three strategies for calculating 35 – 18 + 11.

Grade 3

Original task: 10 x 37
New Task: Find another way to calculate 10 x 37.
New Task +: Find three strategies for calculating 10 x 37 (or 12 x 37).

Grade 4

Original task: 362 ÷ 6
New Task: Find another way to calculate 362 ÷ 6.
New Task +: Find three strategies for calculating 362 ÷ 6 (or 3630 ÷ 20).

Strategy 4: Find another way.

Grade 5

Original task: 8.4 • 1000

New Task: Find another way to calculate 8.4 • 1000.

New Task +: Find at least three strategies for calculating 0.84 • 100,000.

Grade 6

Original task: 18 is what percent of 40?

New Task: Find another way to determine what percent 18 is of 40.

New Task +: Find at least three strategies to determine what percent 18 is of 40 (or 44).

Grade 7

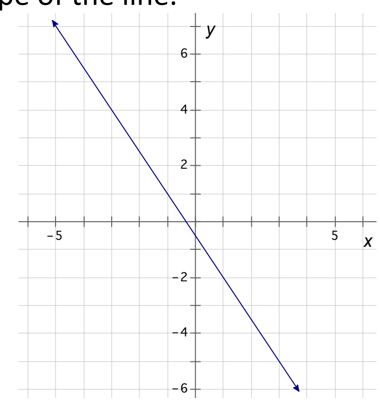
Original task: A shirt that costs 16.50 is on sale for 20% off. What is the sale price?

New Task: Find another strategy to calculate the sale price.

New Task +: Find at least two strategies to calculate the overall percent reduction if the price is reduced by 20%, and then another 20% of that.

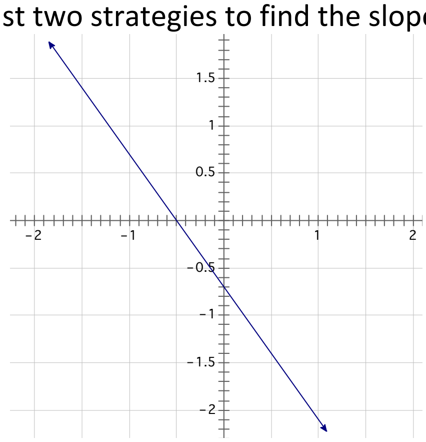
Grade 8

Original task: Find the slope of the line.



New Task: Use another strategy to find the slope of the line.

New Task +: Show at least two strategies to find the slope of this line.



Strategy 5: Compare and contrast.

Opportunities to compare and contrast appear often in math. Students may compare and contrast answers, thinking processes, solution strategies, definitions, geometric figures, etc. This often-overlooked strategy invites students to make connections and distinctions that deepen their understanding of concepts. You may create items for students to compare and contrast, but also watch for opportunities that arise naturally in the course of other tasks!

Grade 1

Original task: True or False? 7 + 4 = 6 + 5

New Task: How are these expressions the same? How are they different?

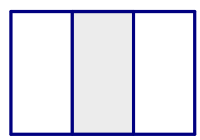
6 + 5   7 + 4   8 + 3   9 + 2

New Task +: How are these expressions the same? How are they different?

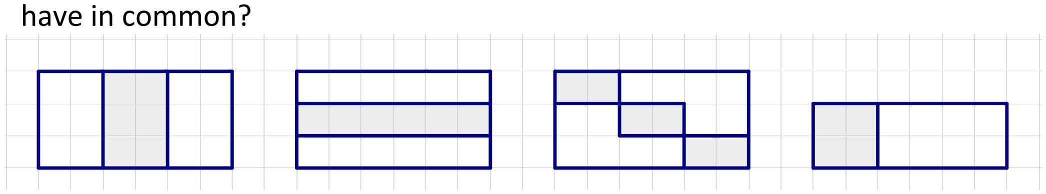
23 + 41   21 + 43   19 + 45   17 + 47

Grade 2

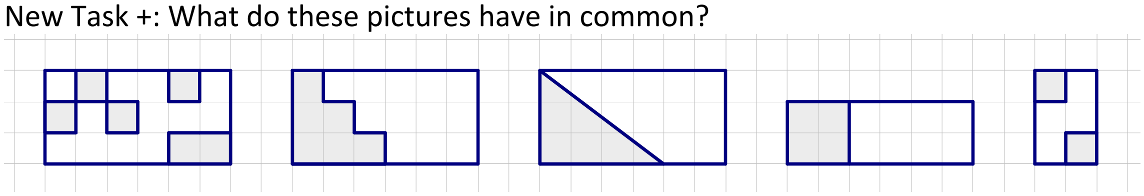
Original task: What fraction does the shaded part show?



New Task: What do the first three pictures have in common? What do they all have in common?



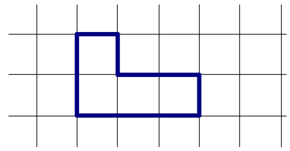
New Task +: What do these pictures have in common?



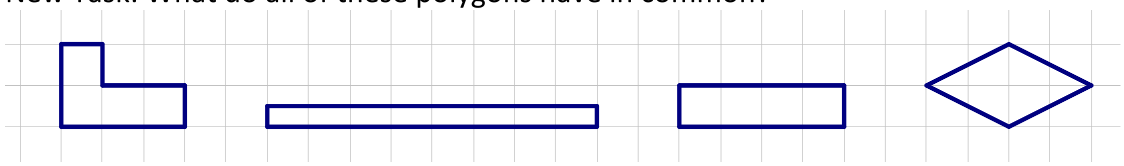
Strategy 5: Compare and contrast.

Grade 3

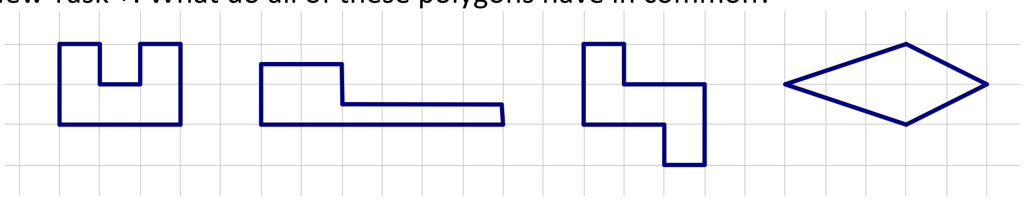
Original task: Find the area.



New Task: What do all of these polygons have in common?



New Task +: What do all of these polygons have in common?



Grade 4

Original task: Find the factors of 42.

New Task: How are the two sets of numbers different? (Think about multiplication.)

First set: 18, 42, 6, 35, 27 Second set: 7, 2, 11, 19, 5

New Task +: How are the two sets of numbers different? (Think about multiplication.)

First set: 10, 143, 8, 22, 6, 77, 95 Second set: 97, 13, 2, 23, 49, 109, 4

Strategy 5: Compare and contrast.

Grade 5

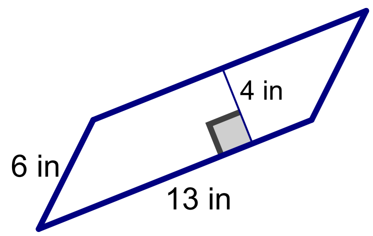
Original task: $6 \cdot \frac{2}{3}$

New Task: Compare and contrast the first set of expressions to the second.

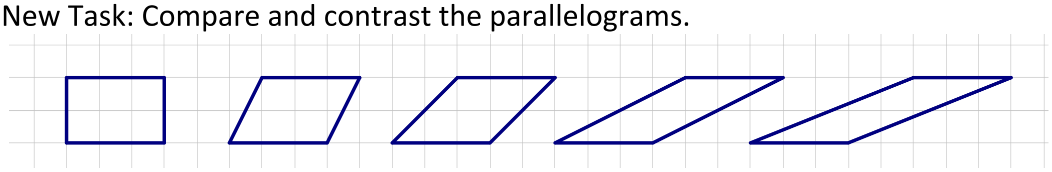
Set 1: $9 \cdot \frac{2}{3}$ $12 \cdot \frac{5}{6}$ $15 \cdot \frac{2}{5}$ Set 2: $6 \cdot \frac{3}{5}$ $7 \cdot \frac{1}{2}$ $12 \cdot \frac{2}{7}$

Grade 6

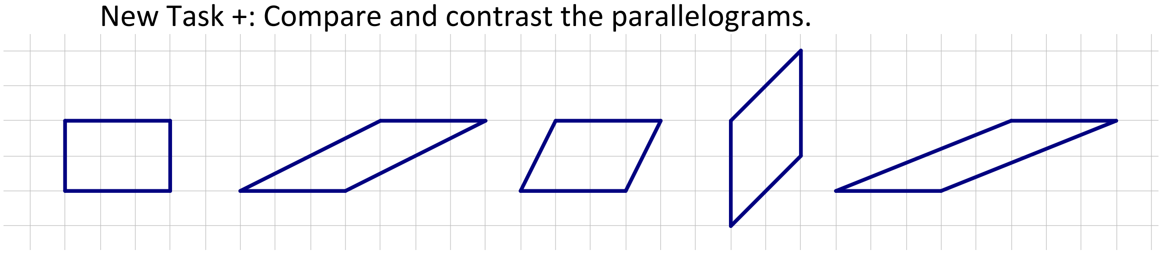
Original task: Find the area of the parallelogram.



New Task: Compare and contrast the parallelograms.



New Task +: Compare and contrast the parallelograms.

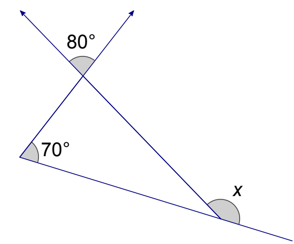


Leaving the question open about what aspects of the figures should be compared and contrasted requires students to learn to identify and attend to important mathematical features. Also, they will often surprise you with their observations, giving you deeper insight into their thinking.

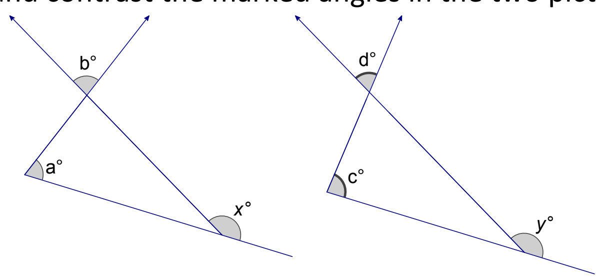
Strategy 5: Compare and contrast.

Grade 7

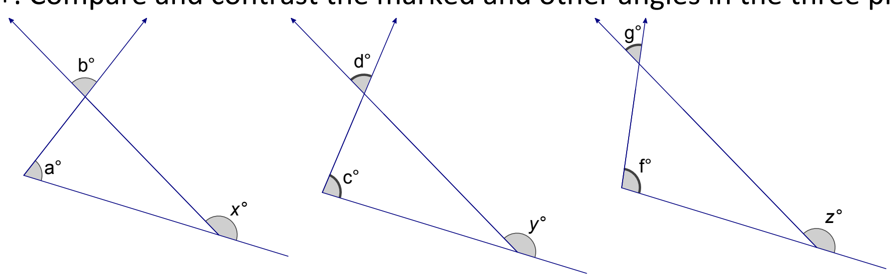
Original task: Find the measure of $\angle x$.



New Task: Compare and contrast the marked angles in the two pictures.



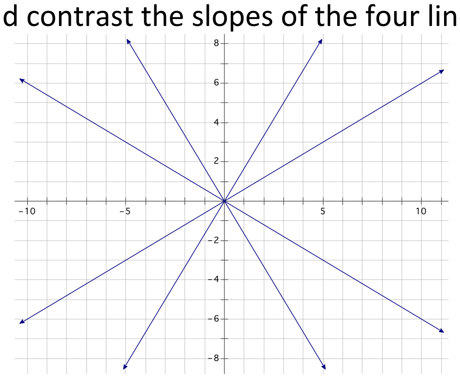
New Task +: Compare and contrast the marked and other angles in the three pictures.



Grade 8

Original task: Find the slope of the line. (See the graph from Strategy 4.)

New Task: Compare and contrast the slopes of the four lines.



New Task +: Use a similar task with axes that show decimal values and/or use perpendicular lines.

Strategy 6: Start with the answer.

Strategy 6 involves thinking backwards. It often leads to problems with multiple solutions, and these solutions often show patterns. When they do, Strategy 6 works well with Strategy 9, Build a Pattern: students organize their solutions to show patterns. They may also use Strategy 3 by explaining what causes the patterns.

Grade 1

Original task: What is the value of the coins?

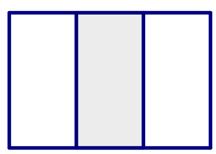
New Task: You have 36¢ in dimes and pennies. How many of each are there?

New Task +: You have $1.23 in dollar bills, dimes, and pennies. How many of each are there?

The New Tasks combine well with Strategy 9: students find multiple answers and organize them to find patterns and make predictions.

Grade 2

Original task: What fraction does the shaded part show?



New Task: The shaded part of a picture shows 1 *third*. What does the picture look like?

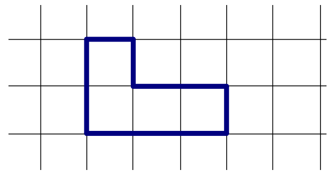
New Task +: Make at least five pictures that show 1 *third*. Make each picture as different as you can.

The New Tasks also make use of Strategy 2, Draw a Picture.

Strategy 6: Start with the answer.

Grade 3

Original task: Find the area.



New Task: The area of a polygon is 4 square units. Draw the polygon.

New Task +: The area of a polygon is 4 square units. Draw at least four examples of the polygon. Make each drawing as different as you can.

The New Tasks also make use of Strategy 2, Draw a Picture.

Grade 4

Original task: 352 ÷ 7

New Task: The quotient is 50 and the remainder is 2. What are the dividend and divisor?

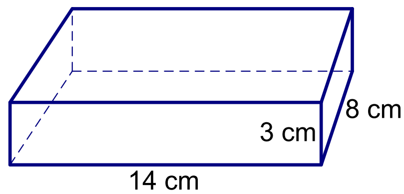
New Task +: The quotient is 50 and the remainder is 2. Show at least four examples of possible dividends and divisors.

The New Tasks combine well with Strategy 9: students find multiple answers and organize them to find patterns and make predictions.

Strategy 6: Start with the answer.

Grade 5

Original task: Find the volume of the rectangular prism.



New Task: The volume of a rectangular prism is 336 cubic cm. Draw a picture of the prism.

New Task +: The volume of a rectangular prism is 336 cubic cm. One of the sides does not have a whole number length. Draw pictures to show at least two different possible prism

Both New Tasks also make use of Strategy 2, Draw a Picture. You could combine the first New Task with Strategy 4 (Find Another Way), Strategy 9 (Build a Pattern), and Strategy 3 (Explain Why) by asking students to find all possible solutions with whole number sides, and use patterns to explain how they know that they have found all of them.

Grade 6

Original task: Find the mean, median, and range. 31, 27, 32, 65, 29

New Task: The mean of five numbers is 36.8, the median is 32, and the range is 38. What are the numbers?

New Task +: The mean of ten numbers is 36.8, the median is 32, and the range is 38. Show at least five possible sets of numbers.

Strategy 6: Start with the answer.

Grade 7

Original Task: You bought an $18.00 shirt on sale for 20% off. How much did you pay?

New Task: You bought a shirt for 20% off, and you paid $14.40. How much did the shirt cost originally?

New Task +: You bought a shirt 20% off. The sales tax was 6%. You paid $15.12. What was the original price of the shirt before the tax and the discount?

Grade 8

Original task: Find the length of the hypotenuse of a right triangle whose legs have lengths of 4 and 10 units.

New Task: A right triangle has a hypotenuse of length $\sqrt{116}$ units. Find the lengths of the legs.

New Task +: Find at least four right triangles that have a hypotenuse of length $\sqrt{116}$. (Alternatively, replace this length by an algebraic expression—even something simple such as *x*.)

Strategy 7: Remove information.

By removing information, you often create problems that have multiple solutions. Strategy 7 may work well with

- Strategy 5: Students may compare and contrast their various solutions.
- Strategy 6: You both remove information *and* start with the answer!
- Strategy 9: Students build patterns from their various answers.

Grade 1

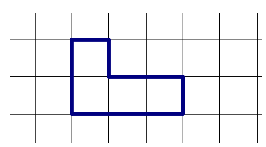
Original task: 13 – 8 = 10 – ____
New Task: ____ – 8 = 10 – ____
New Task +: ____ – 80 = 100 – ____
Students can look for patterns in their answers. (Strategy 9.)

Grade 2

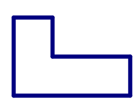
Original task: 273 = ____ ones + ____ tens + ____ hundreds.
New Task: Use numbers and place value words to write 273.
New Task +: Write 2073 in three or more ways using numbers and place value words.

Grade 3

Original task: Find the area.



New Task: Find the area.



New Task +: Compare the two areas.

There are multiple solutions in the New Task, because the area depends on what a square unit looks like. (Some students may draw in their own square units. Others may measure.) The New Task + combines strategies 7 and 5, because students compare areas.

Grade 4

Original task: 84 ÷ 3 = ____
New Task: 84 ÷ ____ = ____
New Task +: 336 ÷ ____ = ____
Students can look for patterns in their answers. (Strategy 9.)

Strategy 7: Remove information.

Grade 5

Original task: $6 \cdot \frac{2}{3}$

New Task: $\square \cdot \frac{\boxed{2}}{\square} = 4$

New Task +: Find at least ten solutions. $\square \cdot \frac{\boxed{3}}{\square} = 4$

Strategies 6 and 7 work well together. You can both remove information *and* start with the answer. Students can also look for patterns in their answers. (Strategy 9.)

In the New Task +, the 3 in the numerator makes the problem more complex, because it is not a factor of 4.

Grade 6

Original task: $6 \div \frac{2}{3}$

New Task: $\square \div \frac{\square}{\boxed{3}} = 9$

New Task +: $\square \div \frac{\square}{\boxed{4}} = 9$

Strategies 6 and 7 work well together. You can both remove information *and* start with the answer. Students can also look for patterns in their answers. (Strategy 9.)

In the New Task +, the 4 in the denominator makes the problem more complex, because it is not a factor of 9.

Strategy 7: Remove information.

Grade 7

Original task: An $18.00 shirt is on sale for 20% off. What is the sale price?

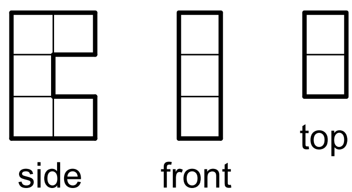
New Task: An $18.00 shirt is on sale for ____% off. What is the sale price?

New Task +: A $15.00 shirt for ____% off costs the same amount as an $18.00 shirt for ____% off.

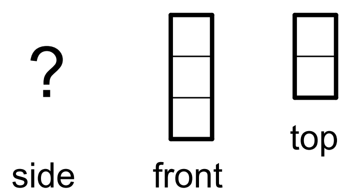
Students can look for patterns in their answers. (Strategy 9.)

Grade 8

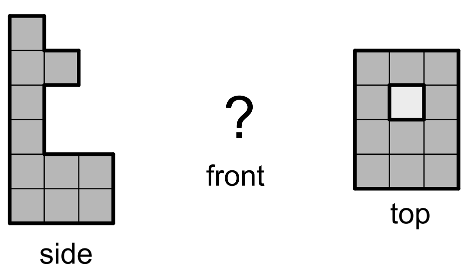
Original task: A figure is made of cubes. Draw the figure.



New Task: A figure is made of cubes.



New Task +: A figure is made of cubes.



Notice that you may even remove the *directions*! Encourage students to come up with their own directions. For example, they may decide either to draw or to make the figure (or both). They may compare and contrast their solutions (Strategy 5), and explain what causes the similarities and differences (Strategy 3).

Strategy 8: Solve to learn.

When you use Strategy 8, the New Task is usually the same as the original task! The difference is that you offer it to students *before* teaching the concept. This turns a basic task into a problem to be solved! Solving to Learn is an excellent approach for all students, but some students may be able to approach the tasks earlier than others and with less support. Before using this strategy, be certain that students are able to make sense of the problem with their current knowledge.

Grade 1

Original task: Write the number that is ten less than 83.

New Task: Write the number that is ten less than 83.

New Task +: Write the number that is twenty less than 116.

Offer manipulatives or visual models (base-ten blocks, 100-boards, etc.) that give students tools to approach the problem. This combines well with Strategy 3.

Grade 2

Original task: 31 – 18

New Task: 31 – 18

New Task +: 31 – 18 – 4

Students can approach this task as a problem if they know what subtraction means and understand place value to tens. Encourage them to search for "easy" ways to solve the problem. Offer or suggest appropriate manipulatives or visual models if necessary.

Grade 3

Original task: 10 x 37

New Task: 10 x 37

New Task +: 100 x 370

Students can approach this task as a problem before learning multi-digit multiplication algorithms or rules about "attaching 0s." Encourage them to search for "easy" ways to solve the problem. Offer or suggest appropriate manipulatives or visual models if necessary. This combines well with Strategy 3, especially if you ask them to use place value ideas to explain.

Grade 4

Original task: Find the factors of 42.

New Task: Find the factors of 42.

New Task +: Find the factors that 42 shares with 60.

Students can approach this task as a problem if they know the definition of a factor and can multiply one-digit numbers. When they do it before learning procedures, they need to think about how to organize their ideas and results so that they know when they have found all of the factors.

Strategy 8: Solve to learn.

Grade 5

Original task: $6 \bullet \frac{2}{3}$

New Task: $6 \bullet \frac{2}{3}$

New Task +: $5 \bullet \frac{2}{3}$

Students can approach this task before learning rules for multiplying fractions by using what they know about the meaning of multiplication. For example, they may think of either “6 groups of 2/3” or “2/3 of a group of 6.” This strategy combines very well with Strategies 1 – 4 in which students create a story, draw a picture, explain why, and find more than one way!

Grade 6

Original task: $6 \div \frac{2}{3}$

New Task: $6 \div \frac{2}{3}$

New Task +: $5 \div \frac{2}{3}$

Students can approach this task before learning rules for dividing fractions by using what they know about the meaning of fractions and division. For example, they may think either “How many groups of 2/3 are in 6?” or “If 6 is 2/3 of a group, how many are in the group?” This strategy combines very well with Strategies 1 – 4 in which they create a story, draw a picture, explain why, and find more than one way!

The New Task + is more complex even though 5 is less than 6, because the answer is not a whole number.

Strategy 8: Solve to learn.

Grade 7

Original task: 13 – -5

New Task: 13 – -5

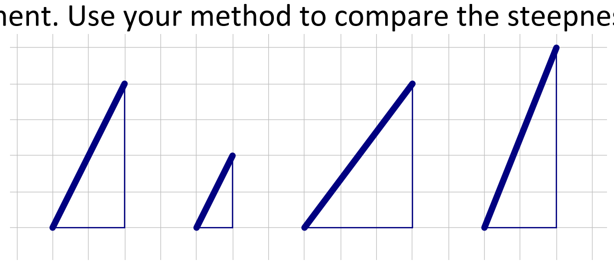
New Task +: -5 – -13 – 6

By giving tasks like this to students before teaching them rules for subtracting positive and negative numbers, they need to apply their own knowledge about the meaning(s) of subtraction to negative numbers in order to develop their own strategies. The can use ideas of (1) take away, (2) how much more, (3) opposite of addition, etc.

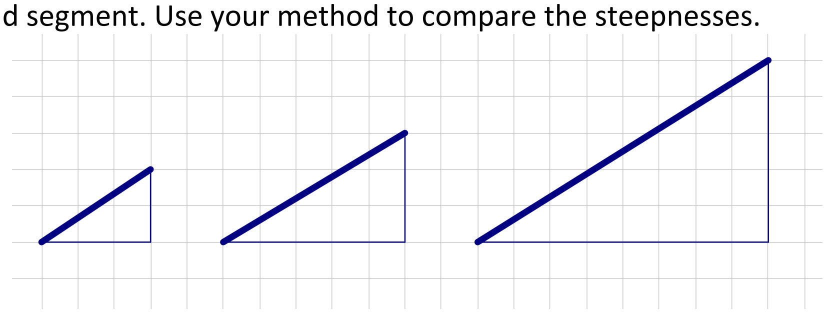
Grade 8

Original task: Find the slope of the line. (It could be any line with a slope that is reasonably simple to describe.)

New Task: Invent a way to calculate a number that describes the steepness of each bold segment. Use your method to compare the steepnesses.



New Task +: Invent a way to use a number to describe the steepness of each bold segment. Use your method to compare the steepnesses.



This is the only example for Strategy 8 in which the new task is different than the original. Still, the basic idea is the same: students invent a method for describing slope *before* they are taught a formula. The light vertical and horizontal segments hint that they should try using these lengths in their calculations. They need to explore different ways of combining the two numbers so that steeper slopes always have larger numbers (and equal slopes have equal numbers)! The New Task + is more complex, because the slopes are hard to distinguish visually.

Strategy 9: Build a pattern.

Patterns are central to mathematics. In fact, some people *define* math to be the study of patterns! In many cases, patterns are a key to understanding properties of numbers and geometric figures. Patterns also help students connect new concepts to old ones by recognizing a new concept as an extension of a pattern learned earlier. For Strategy 9, you may build a pattern and ask students to analyze it and extend it, or you may ask students to build their own patterns (or both).

Grade 1

Original task: True or False? 7 + 4 = 6 + 5

New Task: Describe the pattern and make predictions.

7 + 4   6 + 5   5 + 6   4 + 7   3 + 8   2 + 9

New Task +: Describe the pattern and make predictions.

21 + 16   19 + 18   17 + 20   15 + 22   13 + 24

Grade 2

Original task: 35 – 18

New Task: Describe the patterns and make predictions.

35 – 18   34 – 17   33 – 16   32 – 15   31 – 14   30 – 13

New Task +: Describe the patterns and make predictions.

35 – 18   36 – 16   37 – 14   38 – 12   39 – 10

One of the best ways to deepen students' understanding of an operation is to explore patterns in the ways that an expression's value changes when the expression itself changes in predictable ways.

Grade 3

Original task: Learning facts for a single-digit product times itself

New Task: Describe patterns and make predictions.

2 x 2   3 x 3   4 x 4   5 x 5   6 x 6   7 x 7   8 x 8   9 x 9

1 x 3   2 x 4   3 x 5   4 x 6   5 x 7   6 x 8   7 x 9   8 x 10

New Task +:   Describe patterns and make predictions.

8 x 8   7 x 9   6 x 10   5 x 11   4 x 12   3 x 13

This task could work well in combination with Strategies 2 and 3: students draw pictures (possibly arrays) and use them to explain what causes the patterns.

Grade 4

Original task: Find the factors of 24.

New Task: Find the factors of 3, 6, 12, 24, and 48. Look for patterns. Create new patterns like these.

New Task +: Find and describe patterns in this diagram. Extend the pattern or create your own new pattern like it.

	1	**2**	**4**
1	1	2	4
3	3	6	12
9	9	18	36

Strategy 9: Build a pattern.

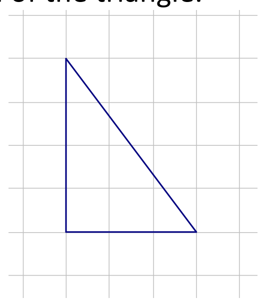
Grade 5

Original task: 8.4 x 1000

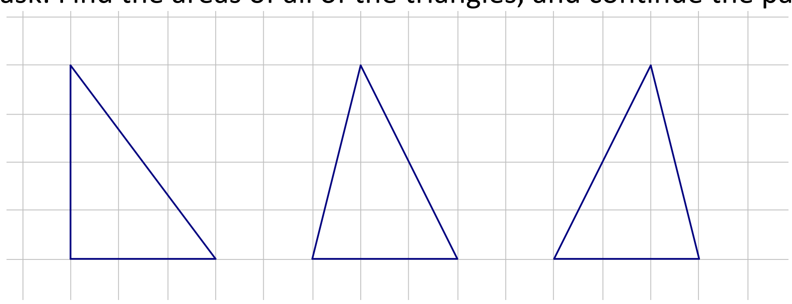
New Task: Describe and extend the pattern. 8.4 x 1000   84 x 100   840 x 10

Grade 6

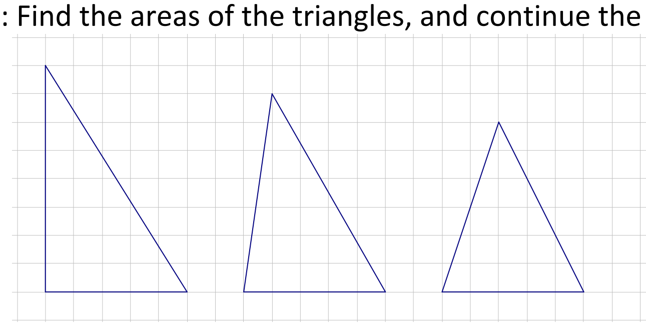
Original task: Find the area of the triangle.



New Task: Find the areas of all of the triangles, and continue the pattern.



New Task +: Find the areas of the triangles, and continue the pattern.



The New Tasks work especially well in combination with Strategy 7 (giving it to students before teaching them a formula) and Strategy 3 in which they explain what causes the patterns.

Strategy 9: Build a pattern.

Grade 7

Original task: Graph each point on a coordinate plane.

(-3, 7)   (-1, 2)   (0,-9)

New Task: Identify and extend the pattern.

(0, 0)	(1, 0)	(1, 1)	(0, 1)
(-1, 1)	(-1, 0)	(-1, -1)	(0, -1)
(1, -1)	(2, -1)	(2, 0)	(2, 1)
(2, 2)	(1, 2)	(0, 2)	(-1, 2)
(-2, 2)	(-2, 1)	(-2, 0)	(-2, -1)

If you read the ordered pairs from the New Task in "book order" (left to right, top to bottom), they look like a spiral when you connect the dots. Some students may be able to identify and extend the pattern without plotting the points, but most students will be more successful if they plot the points.

Grade 8

Original task: Find the value of 3^{-2}.

New Task: Create a pattern to help you find the value of 3^{-2}.

New Task +: Create a pattern to help you find the value of $(0.5)^{-2}$.

The New Tasks may work best if combine them with Strategy 8: Solve to Learn. (Give the tasks to students before you teach them rules for evaluating negative exponents.) A helpful pattern to look at is

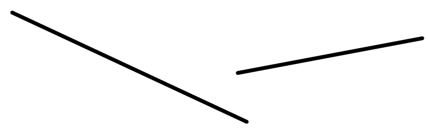
$3^4 = 81 \quad 3^3 = 27 \quad 3^2 = 9 \quad 3^1 = 3$, etc.

Strategy 10: Ask "What if...?"

Strategy 10 is open-ended, because it allows you to ask all sorts of questions about the original task. A common choice is to ask what happens to the answer if you change something in the task. You can make the changes yourself, or you can ask your students to suggest ways to change the task. Often, when you make changes, you see patterns in the results. Because of this, Strategy 10 and Strategy 9 often go well together!

Grade 1

Original task: Which segment is longer?



New Task: What happens if I join two of the long segments together and three of the short segments together? Now which segment is longer?

New Task +: What happens if I use these segments to measure things?

Grade 2

Original task: 35 – 18

New Task: What happens if I make the first number 1 greater? What happens if I make the second number___ greater? What happens if I make both numbers ___ greater?

New Task +: What happens if I make the same changes to other subtraction expressions?

Grade 3

Original task: 47 x 3

New Task: What happens if the tens digit increases by 1? Or some other amount? Or what if the tens digit trades places with the other ones digit?

New Task +: What happens to the product if tens digits and the other ones digit do the same thing in *other* two-digit by one-digit multiplication problems? What about two-digit by two-digit problems?

Grade 4

Original task: Reflect the figure over the line.

(A geometric figure and a line of reflection are shown.)

New Task: What happens if I move the line one unit to the right?

New Task +: What happens if I make a second line and reflect the figure over one line after the other?

Strategy 10: Ask "What if...?"

Grade 5

Original task: $6 \bullet \frac{1}{3}$

New Task: What happens if I double the numerator? The denominator? The whole number? What if I keep doing this?

New Task +: What happens if I halve the numerator? The denominator? The whole number? What if I keep doing these things?

Grade 6

Original task: $6 \div \frac{1}{3}$

New Task: What happens if I double the numerator? The denominator? The whole number? What if I keep doing this?

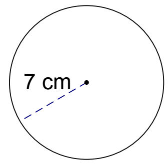
New Task +: What happens if I halve the numerator? The denominator? The whole number? What if I keep doing these things?

You could also answer the same questions for multiplication (see grade 5) and then compare and contrast the results of multiplying vs. dividing.

Strategy 10: Ask "What if...?"

Grade 7

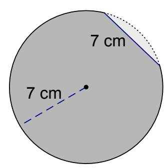
Original task: Find the circumference and area of the circle.



New Task: What happens to the circumference and area if I add 1 to the radius? What happens if I double the radius?

This task would work well with Strategy 3: explain why these things happen.

Alternate New Task: What happens to the area and circumference if I move the radius to make a chord and then "chop off" the small region that it forms?



Students may or may not have the knowledge needed to answer this question. This may happen sometimes, especially when *they* create the new tasks. Students can still try to solve the problem using what they know. The experience of figuring out what knowledge they are lacking is valuable in itself! They may even solve the problem using creative "workarounds." Estimation and other strategies may come into play.

Grade 8

Original task: Solve the equation.

$$5(x - 3) = 12$$

New Task: What happens to the solution if I increase 3 by 1? What if I keep increasing it by 1? What if I start with the 5 and keep increasing it by 1?
New Task +: Make similar types of changes to a more complex equation.

This New Task works very well with strategies 9 and 3, because students can look for patterns in the way that the solutions change and explain what causes them.

Ten Plus One:

Strategies for Enhancing Depth and Complexity of Math Tasks

examples organized by grade

by Jerry Burkhart

Grade 1

Strategy 1: Write a story.

Original task: 36 + 7

New Task: Write and answer a story problem for 36 + 7.

New Task +: Write and answer a story problem for 36 + 27.

Strategy 2: Draw a picture.

Original task: True or False? 7 + 4 = 6 + 5

New Task: Draw a picture or diagram to show that 7 + 4 is equal to 6 + 5.

New Task +: Draw a picture or diagram to show that 7 + 4 + 9 is equal to 9 + 6 + 5.

Strategy 3: Explain why.

Original task: True or False: 7 + 4 = 6 + 5

New Task: Explain why 7 + 4 = 6 + 5 without finding the sums.

New Task +: Explain why 7 + 4 + 9 is equal to 9 + 6 + 5 without find the sums.

Strategy 4: Find another way.

Original task: 36 + 7

New Task: Find another way to calculate 36 + 7.

New Task +: Find three ways to calculate 36 +27.

Grade 1

Strategy 5: Compare and contrast.

Original task: True or False? 7 + 4 = 6 + 5

New Task: How are these expressions the same? How are they different?

6 + 5   7 + 4   8 + 3   9 + 2

New Task +: How are these expressions the same? How are they different?

23 + 41   21 + 43   19 + 45   17 + 47

Strategy 6: Start with the answer.

Original task: What is the value of the coins?

New Task: You have 36¢ in dimes and pennies. How many of each are there?

New Task +: You have $1.23 in dollar bills, dimes, and pennies. How many of each are there?

The New Tasks combine well with Strategy 9: students find multiple answers and organize them to find patterns and make predictions.

Strategy 7: Remove information.

Original task: 13 – 8 = 10 – ____

New Task: ____ – 8 = 10 – ____

New Task +: ____ – 80 = 100 – ____

Students can also look for patterns in their answers. (Strategy 9.)

Strategy 8: Solve to learn.

Original task: Write the number that is ten less than 83.

New Task: Write the number that is ten less than 83.

New Task +: Write the number that is twenty less than 116.

Offer manipulatives or visual models (base-ten blocks, 100-boards, etc.) that give students tools to approach the problem. This combines well with Strategy 3: Explain why).

Grade 1

Strategy 9: Build a pattern.

Original task: True or False? 7 + 4 = 6 + 5

New Task: Describe the pattern and make predictions.

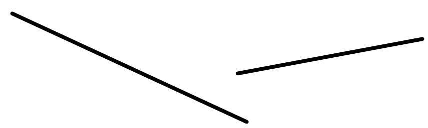
7 + 4   6 + 5   5 + 6   4 + 7   3 + 8   2 + 9

New Task +: Describe the pattern and make predictions.

21 + 16   19 + 18   17 + 20   15 + 22   13 + 24

Strategy 10: Ask "What if…?"

Original task: Which segment is longer?



New Task: What happens if I join two of the long segments together and three of the short segments together? Now which segment is longer?

New Task +: What happens if I use these segments to measure things?

Grade 2

Strategy 1: Write a story.

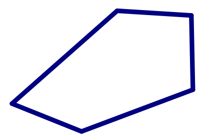
Original task: 35 – 18

New Task: Write and answer a story problem for 35 – 18.

New Task +: Write and answer a story problem for 35 – 18 + 11.

Strategy 2: Draw a picture.

Original task: Name the shape.



New Task: Draw a pentagon whose sides are all different lengths.

New Task +: Draw five pentagons, and make each one as different as you can.

Asking students to make each pentagon as different as possible stretches them to test the limits of what it means for something to be a pentagon.

Strategy 3: Explain why.

Original task: Write <, >, or =.   302 ____ 297

New Task: Write <, >, or =. 302____ 297. Explain why your answer makes sense.

New Task +: Explain why 302 – 14 < 297 – 8.

Strategy 4: Find another way.

Original task: 35 – 18

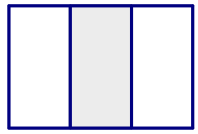
New Task: Find another way to calculate 35 – 18.

New Task +: Find three strategies for calculating 35 – 18 + 11.

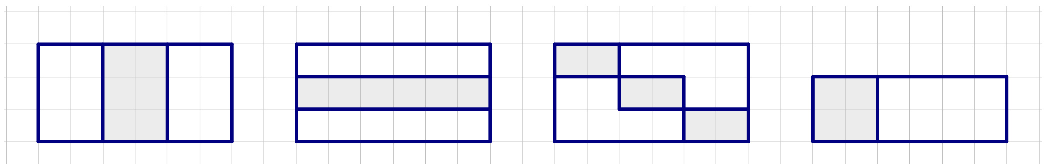
Grade 2

Strategy 5: Compare and contrast.

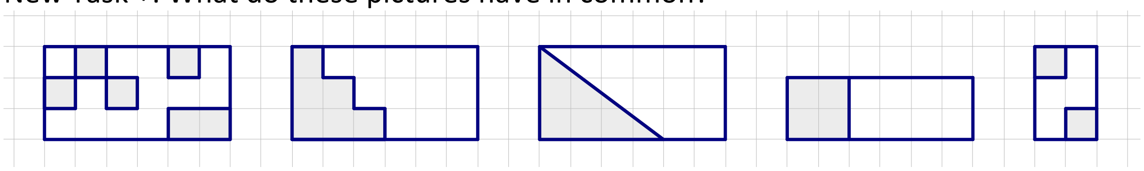
Original task: What fraction does the shaded part show?



New Task: What do the first three pictures have in common? What do they all have in common?

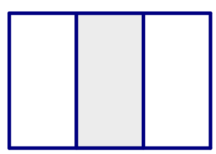


New Task +: What do these pictures have in common?



Strategy 6: Start with the answer.

Original task: What fraction does the shaded part show?



New Task: The shaded part of a picture shows 1 *third*. What does the picture look like?

New Task +: Make at least five pictures that show 1 *third*. Make each picture as different as you can.

The New Tasks also make use of Strategy 2, Draw a Picture.

Grade 2

Strategy 7: Remove information.

Original task: 273 = ____ ones + ____ tens + ____ hundreds.

New Task: Use numbers and place value words to write 273.

New Task +: Write 2073 in three or more ways using numbers and place value words.

Strategy 8: Solve to learn.

Original task: 31 – 18

New Task: 31 – 18

New Task +: 31 – 18 – 4

Students can approach this task as a problem if they know what subtraction means and understand place value to tens. Encourage them to search for "easy" ways to solve the problem. Offer or suggest appropriate manipulatives or visual models if necessary.

Strategy 9: Build a pattern.

Original task: 35 – 18

New Task: Describe the patterns and make predictions.

35 – 18 34 – 17 33 – 16 32 – 15 31 – 14 30 – 13

New Task +: Describe the patterns and make predictions.

35 – 18 36 – 16 37 – 14 38 – 12 39 – 10

One of the best ways to deepen students' understanding of an operation is to explore patterns in the ways that an expression's value changes when the expression itself changes in predictable ways.

Strategy 10: Ask "What if…?"

Original task: 35 – 18

New Task: What happens if I make the first number 1 greater? What happens if I make the second number___ greater? What happens if I make both numbers ___ greater?

New Task +: What happens if I make the same changes to other subtraction expressions?

Grade 3

Strategy 1: Write a story.

Original task: 10 x 37

New Task: Write and answer a real-world story problem for 10 x 37.

New Task +: Write and answer a real-world story problem for 10 x 10 x 37.

Strategy 2: Draw a picture.

Original task: Fill in the box to make the fractions equivalent. $\frac{2}{3} = \frac{6}{\square}$

New Task: Draw a diagram to show that $\frac{2}{3}$ is equivalent to $\frac{6}{9}$.

New Task +: Draw a diagram to show that $\frac{6}{8}$ is equivalent to $\frac{9}{12}$.

The New Task + is more complex because neither fraction is in simplest form.

Strategy 3: Explain why.

Original task: 10 x 37

New Task: Explain why the product of 10 x 37 looks like 37 followed by 0.

New Task +: Explain why the product of 100 x 37 looks like 37 followed by two 0s.

Strategy 4: Find another way.

Original task: 10 x 37

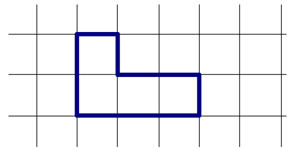
New Task: Find another way to calculate 10 x 37.

New Task +: Find three strategies for calculating 10 x 37 (or 12 x 37).

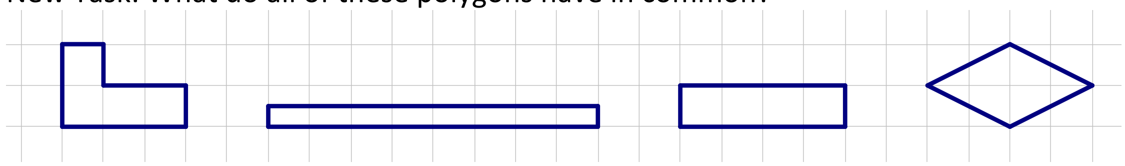
Grade 3

Strategy 5: Compare and contrast.

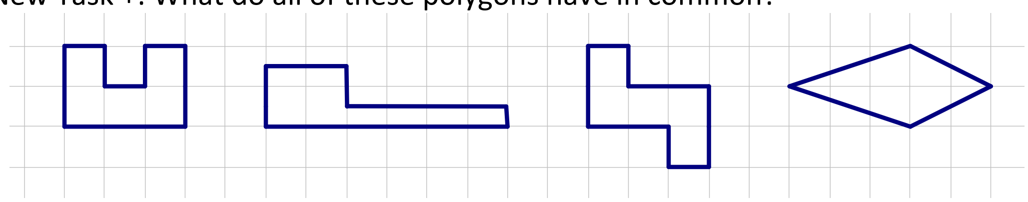
Original task: Find the area.



New Task: What do all of these polygons have in common?

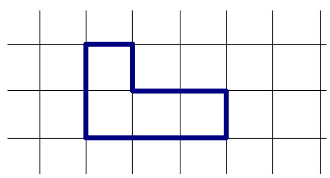


New Task +: What do all of these polygons have in common?



Strategy 6: Start with the answer.

Original task: Find the area.



New Task: The area of a polygon is 4 square units. Draw the polygon.

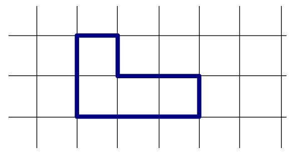
New Task +: The area of a polygon is 4 square units. Draw at least four examples of the polygon. Make each drawing as different as you can.

The New Tasks also make use of Strategy 2, Draw a Picture.

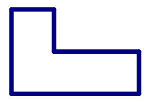
Grade 3

Strategy 7: Remove information.

Original task: Find the area.



New Task: Find the area.



New Task +: Compare the two areas.

There are multiple solutions in the New Task, because the area depends on what a square unit looks like. (Some students may draw in their own square units. Others may measure.) The New Task + combines strategies 7 and 5, because students compare areas.

Strategy 8: Solve to learn.

Original task: 10 x 37

New Task: 10 x 37

New Task +: 100 x 370

Students can approach this task as a problem before learning multi-digit multiplication algorithms or rules about "attaching 0s." Encourage them to search for "easy" ways to solve the problem. Offer or suggest appropriate manipulatives or visual models if necessary. This combines well with Strategy 3, especially if you ask them to use place value ideas to explain

Grade 3

Strategy 9: Build a pattern.

Original task: Learn multiplication facts: a single-digit product times itself.

New Task: Describe patterns and make predictions.

2 x 2   3 x 3   4 x 4   5 x 5   6 x 6   7 x 7   8 x 8   9 x 9

1 x 3   2 x 4   3 x 5   4 x 6   5 x 7   6 x 8   7 x 9   8 x 10

New Task +: Describe patterns and make predictions.

8 x 8   7 x 9   6 x 10   5 x 11   4 x 12   3 x 13

This task could work well in combination with Strategies 2 and 3: students draw pictures (possibly arrays) and use them to explain what causes the patterns.

Strategy 10: Ask "What if…?"

Original task: 47 x 3

New Task: What happens if the tens digit increases by 1? Or some other amount? Or what if the tens digit trades places with the other ones digit?

New Task +: What happens to the product if tens digits and the other ones digit do the same thing in *other* two-digit by one-digit multiplication problems? What about two-digit by two-digit problems?

Grade 4

Strategy 1: Write a story.

Original task: 362 ÷ 6

New Task: Write and answer a real-world story problem for 362 ÷ 6.

New Task +: Write and answer a real-world story problem for 362 ÷ (6 x 2).

Strategy 2: Draw a diagram.

Original task: 62 ÷ 6

New Task: Draw a diagram showing the meaning of 62 ÷ 6 and the answer.

New Task +: Draw diagrams showing two meanings of 62 ÷ 6 and the answer.

Division expressions always have two meanings based on the idea of groups. In this case, the meanings are "If you divide 62 into 6 equal groups, how many are in each group?" and "How groups of 6 are in 62?" You can also use division to compare two numbers: "62 is how many times as much as 6?" In these cases, the answer is not a whole number, so students will have to account for the remainder.

Strategy 3: Explain why.

Original task: $\frac{5}{8}-\frac{1}{4}$

New Task: Explain why $\frac{5}{8}-\frac{1}{4}\neq\frac{5-1}{8-4}$.

New Task +: Explain why $\frac{7}{12}-\frac{4}{8}\neq\frac{7-4}{12-8}$.

Sometimes, explaining why *not* offers a more approachable way to get at a deep concept.

Strategy 4: Find another way.

Original task: 362 ÷ 6

New Task: Find another way to calculate 362 ÷ 6.

New Task +: Find three strategies for calculating 362 ÷ 6 (or 3630 ÷ 20).

Grade 4

Strategy 5: Compare and contrast.

Original task: Find the factors of 42.

New Task: How are the two sets of numbers different? (Think about multiplication.)

First set: 18, 42, 6, 35, 27 Second set: 7, 2, 11, 19, 5

New Task +: How are the two sets of numbers different? (Think about multiplication.)

First set: 10, 143, 8, 22, 6, 77, 95 Second set: 97, 13, 2, 23, 49, 109, 4

Strategy 6: Start with the answer.

Original task: 352 ÷ 7

New Task: The quotient is 50 and the remainder is 2. What are the dividend and divisor?

New Task +: The quotient is 50 and the remainder is 2. Show at least four examples of possible dividends and divisors.

The New Tasks combine well with Strategy 9: students find multiple answers and organize them to find patterns and make predictions.

Strategy 7: Remove information.

Original task: 84 ÷ 3 = ____

New Task: 84 ÷ ____ = ____

New Task +: 336 ÷ ____ = ____

Students can look for patterns in their answers. (Strategy 9.)

Strategy 8: Solve to learn

Original task: Find the factors of 42.

New Task: Find the factors of 42.

New Task +: Find the factors that 42 shares with 60.

Students can approach this task as a problem if they know the definition of a factor and can multiply one-digit numbers. When they do it before learning procedures, they need to think about how to organize their ideas and results so that they know when they have found all of the factors.

Grade 4

Strategy 9: Build a pattern.

Original task: Find the factors of 24.

New Task: Find the factors of 3, 6, 12, 24, and 48. Look for patterns. Create new patterns like these.

New Task +: Find and describe patterns in this diagram. Extend the pattern or create your own new pattern like it.

	1	**2**	**4**
1	1	2	4
3	3	6	12
9	9	18	36

Strategy 10: Ask “What if…?”

Original task: Reflect the figure over the line.

(A geometric figure and a line of reflection are shown.)

New Task: What happens if I move the line one unit to the right?

New Task +: What happens if I make a second line and reflect the figure over one line after the other?

Grade 5

Strategy 1: Write a story.

Original task: Write >, <, or =.   0.7 ___ 0.58

New Task: Write a real-world story about comparing 0.7 and 0.58.

New Task +: Write a real-world story about comparing 0.07, 0.058, and 0.12.

Strategy 2: Draw a diagram.

Original task: $6 \cdot \frac{2}{3}$

New Task: Draw a diagram showing the meaning and the value of $6 \cdot \frac{2}{3}$.

New Task +: Draw diagrams showing two meanings and the value of $6 \cdot \frac{2}{3}$ (or $5 \cdot \frac{2}{3}$).

Two important meanings are “6 groups of 2/3” and “2/3 of a group of 6.”

Strategy 3: Explain why.

Original task: $6 \cdot \frac{2}{3}$

New Task: Explain why $6 \cdot \frac{2}{3} = 4$.

New Task +: Explain why $6 \cdot \frac{3}{4} = 4\frac{1}{2}$.

Strategy 4: Find another way.

Original task: 8.4 • 1000

New Task: Find another way to calculate 8.4 • 1000.

New Task +: Find at least three strategies for calculating 0.84 • 100,000.

Grade 5

Strategy 5: Compare and contrast.

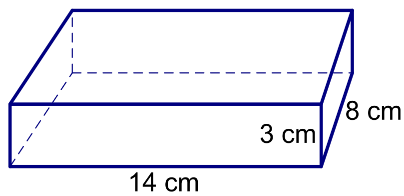
Original task: $6 \cdot \frac{2}{3}$

New Task: Compare and contrast the first set of expressions to the second.

Set 1: $9 \cdot \frac{2}{3}$ $\quad 12 \cdot \frac{5}{6}$ $\quad 15 \cdot \frac{2}{5}$ Set 2: $6 \cdot \frac{3}{5}$ $\quad 7 \cdot \frac{1}{2}$ $\quad 12 \cdot \frac{2}{7}$

Strategy 6: Start with the answer.

Original task: Find the volume of the rectangular prism.



New Task: The volume of a rectangular prism is 336 cubic cm. Draw a picture of the prism.

New Task +: The volume of a rectangular prism is 336 cubic cm. One of the sides does not have a whole number length. Draw pictures to show at least two different possible prism

Both New Tasks also make use of Strategy 2, Draw a Picture. You could combine the first New Task with Strategy 4 (Find Another Way), Strategy 9 (Build a Pattern), and Strategy 3 (Explain Why) by asking students to find all possible solutions with whole number sides, and use patterns to explain how they know that they have found all of them.

Strategy 7: Remove information.

Original task: $6 \cdot \frac{2}{3}$

New Task: $\square \cdot \frac{\boxed{2}}{\square} = 4$

New Task +: Find at least ten solutions. $\square \cdot \frac{\boxed{3}}{\square} = 4$

Strategies 6 and 7 work well together. You can both remove information *and* start with the answer. Students can also look for patterns in their answers. (Strategy 9.)

In the New Task +, the 3 in the numerator makes the problem more complex, because it is not a factor of 4.

Grade 5

Strategy 8: Solve to learn.

Original task: $6 \bullet \frac{2}{3}$

New Task: $6 \bullet \frac{2}{3}$

New Task +: $5 \bullet \frac{2}{3}$

Students can approach this task before learning rules for multiplying fractions by using what they know about the meaning of multiplication. For example, they may think of either "6 groups of 2/3" or "2/3 of a group of 6." This strategy combines very well with Strategies 1 – 4 in which students create a story, draw a picture, explain why, and find more than one way!

Strategy 9: Build a pattern.

Original task: 8.4 x 1000

New Task: Describe and extend the pattern. 8.4 x 1000   84 x 100   840 x 10

Strategy 10: Ask "What if…?"

Original task: $6 \bullet \frac{1}{3}$

New Task: What happens if I double the numerator? The denominator? The whole number? What if I keep doing this?

New Task +: What happens if I halve the numerator? The denominator? The whole number? What if I keep doing these things?

Grade 6

Strategy 1: Write a story.

Original task: $6 \div \frac{2}{3}$

New Task: Write and answer a real-world story problem for $6 \div \frac{2}{3}$.

New Task +: Write and answer a real-world story problem for $5 \div \frac{2}{3}$.

Strategy 2: Draw a picture.

Original task: $6 \div \frac{2}{3}$

New Task: Draw a diagram showing the meaning and the value of $6 \div \frac{2}{3}$.

New Task +: Draw diagrams showing two meanings and the value of $6 \div \frac{2}{3}$ (or $6 \div \frac{4}{5}$).

Two important meanings are "number of groups of 2/3 in 6" and "the size of one (whole) group is 6 is 2/3 of a group. Some students may think of the inverse of multiplication: "What must I multiply 2/3 by to get 6?" or "2/3 or what is 6?"

Strategy 3: Explain why.

Original task: $6 \div \frac{2}{3}$

New Task: Explain why $6 \div \frac{2}{3} = 9$.

New Task +: Explain why $6 \div \frac{4}{5} = 7\frac{1}{2}$.

Strategy 4: Find another way.

Original task: 18 is what percent of 40?

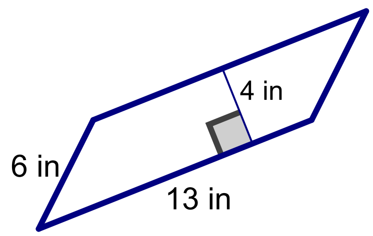
New Task: Find another way to determine what percent 18 is of 40.

New Task +: Find at least three strategies to determine what percent 18 is of 40 (or 44).

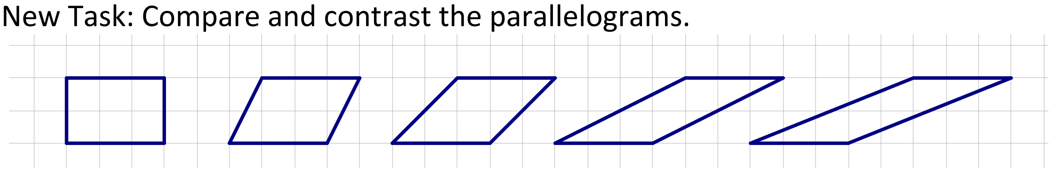
Grade 6

Strategy 5: Compare and contrast.

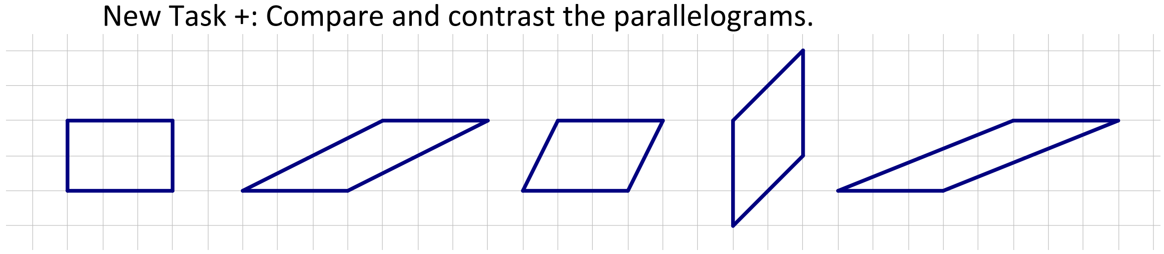
Original task: Find the area of the parallelogram.



New Task: Compare and contrast the parallelograms.



New Task +: Compare and contrast the parallelograms.



Leaving the question open about what aspects of the figures should be compared and contrasted requires students to learn to identify and attend to important mathematical features. Also, they will often surprise you with their observations, giving you deeper insight into their thinking.

Strategy 6: Start with the answer.

Original task: Find the mean, median, and range. 31, 27, 32, 65, 29

New Task: The mean of five numbers is 36.8, the median is 32, and the range is 38. What are the numbers?

New Task +: The mean of ten numbers is 36.8, the median is 32, and the range is 38. Show at least five possible sets of numbers.

Grade 6

Strategy 7: Remove information.

Original task: $6 \div \frac{2}{3}$

New Task: $\square \div \frac{\square}{\boxed{3}} = 9$

New Task +: $\square \div \frac{\square}{\boxed{4}} = 9$

Strategies 6 and 7 work well together. You can both remove information *and* start with the answer. Students can also look for patterns in their answers. (Strategy 9.)

In the New Task +, the 4 in the denominator makes the problem more complex, because it is not a factor of 9.

Strategy 8: Solve to learn.

Original task: $6 \div \frac{2}{3}$

New Task: $6 \div \frac{2}{3}$

New Task +: $5 \div \frac{2}{3}$

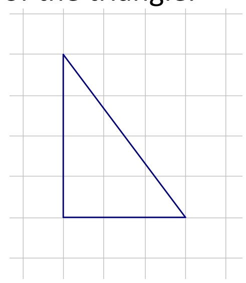
Students can approach this task before learning rules for dividing fractions by using what they know about the meaning of fractions and division. For example, they may think either “How many groups of 2/3 are in 6?” or “If 6 is 2/3 of a group, how many are in the group?” This strategy combines very well with Strategies 1 – 4 in which they create a story, draw a picture, explain why, and find more than one way!

The New Task + is more complex even though 5 is less than 6, because the answer is not a whole number.

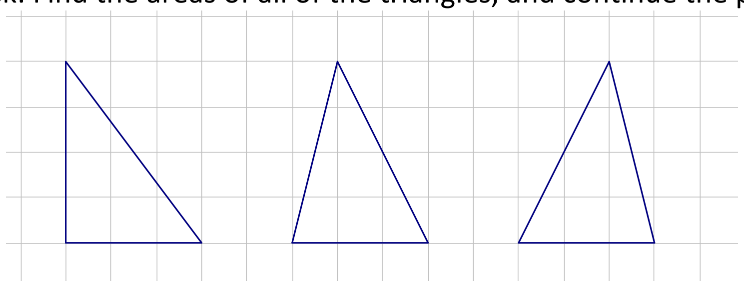
Grade 6

Strategy 9: Build a pattern.

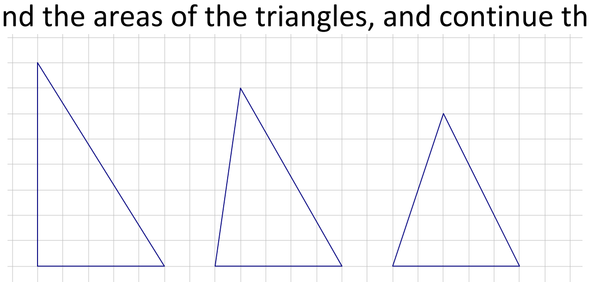
Original task: Find the area of the triangle.



New Task: Find the areas of all of the triangles, and continue the pattern.



New Task +: Find the areas of the triangles, and continue the pattern.



The New Tasks work especially well in combination with Strategy 7 (giving it to students before teaching them a formula) and Strategy 3 in which they explain what causes the patterns.

Strategy 10: Ask "What if…?"

Original task: $6 \div \frac{1}{3}$

New Task: What happens if I double the numerator? The denominator? The whole number? What if I keep doing this?

New Task +: What happens if I halve the numerator? The denominator? The whole number? What if I keep doing these things?

You could also answer the same questions for multiplication (see grade 5) and then compare and contrast the results of multiplying vs. dividing.

Grade 7

Strategy 1: Write a story.

Original task: 13 – -5

New Task: Write and answer a real-world story problem for 13 – -5.

New Task +: Write and answer two real-world story problems for 13 – -5.

Use a different meaning of subtraction and a different real-world situation for each story.

Strategy 2: Draw a picture.

Original task: 13 – -5

New Task: Draw a diagram showing the meaning and the value of 13 – -5.

New Task +: Draw two models that show different ways to understand the meaning of 13 – -5 and how to find its value.

Students usually show a number line diagram (indicating how far and in which direction to travel) or the "take away" meaning. They may also show a comparison ("how much more?") meaning. There are many ways to think about subtraction with negative numbers!

Strategy 3: Explain why.

Original task: 13 – -5

New Task: Explain why 13 – -5 > 13.

New Task +: Explain why -13 – -5 – 6 is one less than -13.

Strategy 4: Find another way.

Original task: A shirt that costs 16.50 is on sale for 20% off. What is the sale price?

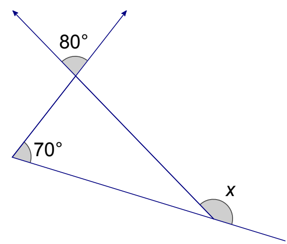
New Task: Find another strategy to calculate the sale price.

New Task +: Find at least two strategies to calculate the overall percent reduction if the price is reduced by 20%, and then another 20% of that.

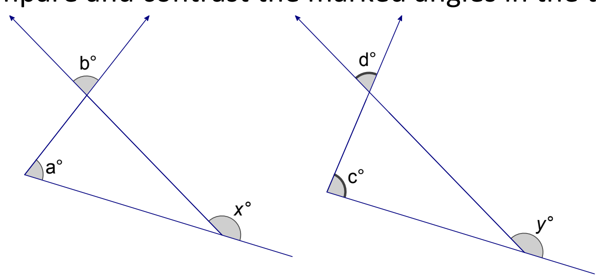
Grade 7

Strategy 5: Compare and contrast.

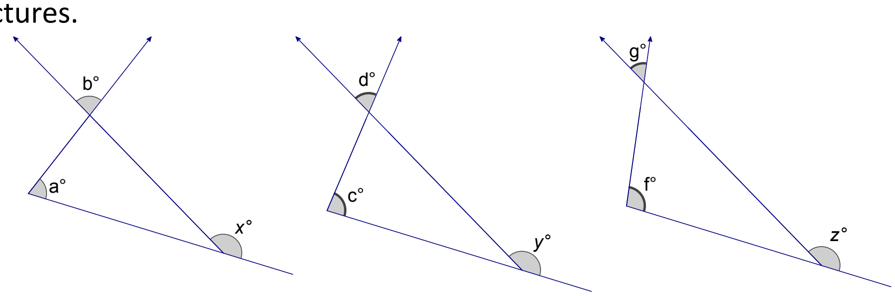
Original task: Find the measure of $\angle x$.



New Task: Compare and contrast the marked angles in the two pictures.



New Task +: Compare and contrast the marked and other angles in the three pictures.



Strategy 6: Start with the answer.

Original Task: You bought an $18.00 shirt on sale for 20% off. How much did you pay?

New Task: You bought a shirt for 20% off, and you paid $14.40. How much did the shirt cost originally?

New Task +: You bought a shirt 20% off. The sales tax was 6%. You paid $15.12. What was the original price of the shirt before the tax and the discount?

Grade 7

Strategy 7: Remove information.

Original task: An $18.00 shirt is on sale for 20% off. What is the sale price?

New Task: An $18.00 shirt is on sale for ____% off. What is the sale price?

New Task +: A $15.00 shirt for ____% off costs the same amount as an $18.00 shirt for ____% off.

Students can look for patterns in their answers. (Strategy 9.)

Strategy 8: Solve to learn.

Original task: 13 – -5
New Task: 13 – -5
New Task +: -5 – -13 – 6

By giving tasks like this to students before teaching them rules for subtracting positive and negative numbers, they need to apply their own knowledge about the meaning(s) of subtraction to negative numbers in order to develop their own strategies. The can use ideas of (1) take away, (2) how much more, (3) opposite of addition, etc.

Grade 7

Strategy 9: Build a pattern.

Original task: Graph each point on a coordinate plane.

(-3, 7) (-1, 2) (0,-9)

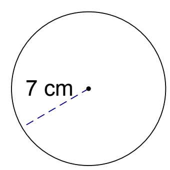
New Task: Identify and extend the pattern.

(0, 0)	(1, 0)	(1, 1)	(0, 1)
(-1, 1)	(-1, 0)	(-1, -1)	(0, -1)
(1, -1)	(2, -1)	(2, 0)	(2, 1)
(2, 2)	(1, 2)	(0, 2)	(-1, 2)
(-2, 2)	(-2, 1)	(-2, 0)	(-2, -1)

If you read the ordered pairs from the New Task in "book order" (left to right, top to bottom), they look like a spiral when you connect the dots. Some students may be able to identify and extend the pattern without plotting the points, but most students will be more successful if they plot the points.

Strategy 10: Ask "What if…?"

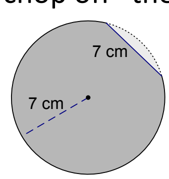
Original task: Find the circumference and area of the circle.



New Task: What happens to the circumference and area if I add 1 to the radius? What happens if I double the radius?

This task would work well with Strategy 3: explain why these things happen.

Alternate New Task: What happens to the area and circumference if I move the radius to make a chord and then "chop off" the small region that it forms?

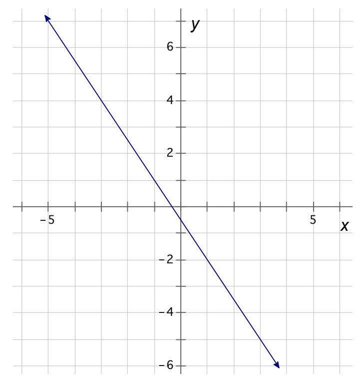


Students may or may not have the knowledge needed to answer this question. This may happen sometimes, especially when *they* create the new tasks. Students can still try to solve the problem using what they know. The experience of figuring out what knowledge they are lacking is valuable in itself! They may even solve the problem using creative "workarounds." Estimation and other strategies may come into play.

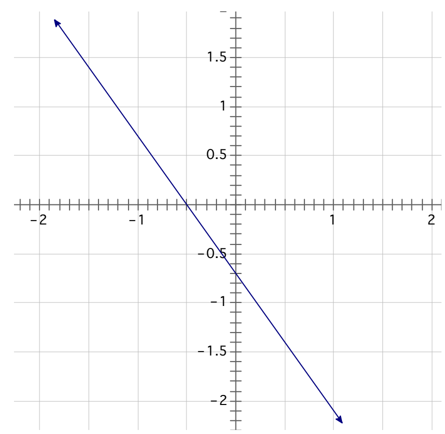
Grade 8

Strategy 1: Write a story.

Original task: Find the slope of the line.



New Task: Write a real-world story about The line and its slope. Tell what *x, y,* and the slope represent.



New Task +: Write a real-world story about the line and its slope. Tell what *x*, *y*, and the slope represent.

Grade 8

Strategy 2: Draw a picture.

Original task: Find the length of the hypotenuse of a right triangle with legs of 4 and 10 units.

New Task: On graph paper, draw a right triangle with legs of 4 units and 10 units. Draw a square on the hypotenuse and use it to find the length of the hypotenuse.

New Task +: On graph paper, draw a right triangle with legs of 4 units and 10.5 units. Draw a square on the hypotenuse and use it to find the length of the hypotenuse.

The New Tasks work well even if (or especially if?) students have not yet been taught the Pythagorean theorem. If you give the task to them before teaching the theorem, then you are also using Strategy 7: Solve to Learn.

Strategy 3: Explain why.

Original task: Decide if each number is rational or irrational:

$$\frac{3}{4} \quad \sqrt{10} \quad \frac{2.3}{0.8} \quad \pi \quad 0 \quad -6 \quad 2\cdot\sqrt{9}$$

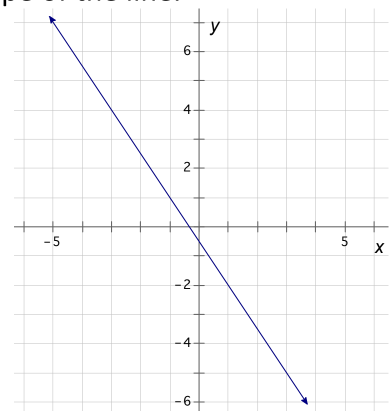
New Task: Explain why $\frac{2.3}{0.8}$ is a rational number.

New Task +: Explain why $\frac{2.3-\frac{3}{4}}{0.8}$ is a rational number.

Grade 8

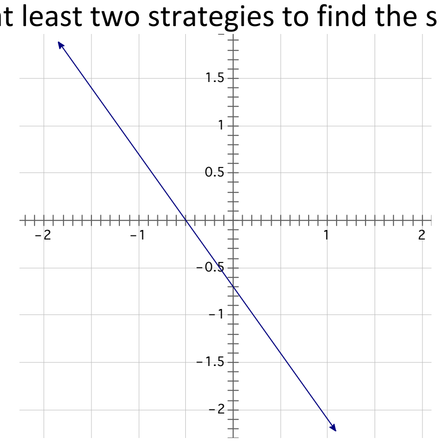
Strategy 4: Find another way.

Original task: Find the slope of the line.



New Task: Use another strategy to find the slope of the line.

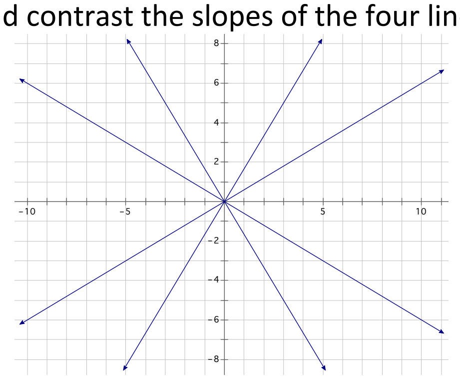
New Task +: Show at least two strategies to find the slope of this line.



Strategy 5: Compare and contrast.

Original task: Find the slope of the line. (See the graph from Strategy 4.)

New Task: Compare and contrast the slopes of the four lines.



New Task +: Use a similar task with axes that show decimal values and/or use perpendicular lines.

Grade 8

Strategy 6: Start with the answer.

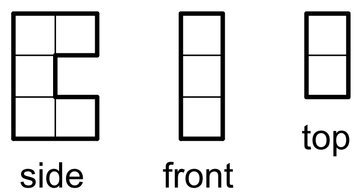
Original task: Find the length of the hypotenuse of a right triangle whose legs have lengths of 4 and 10 units.

New Task: A right triangle has a hypotenuse of length $\sqrt{116}$ units. Find the lengths of the legs.

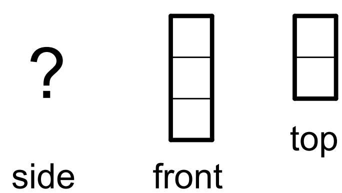
New Task +: Find at least four right triangles that have a hypotenuse of length $\sqrt{116}$. (Alternatively, replace this length by an algebraic expression—even something simple such as *x*.)

Strategy 7: Remove information.

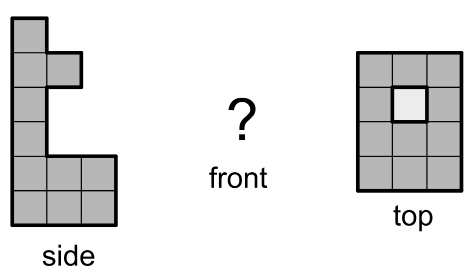
Original task: A figure is made of cubes. Draw the figure.



New Task: A figure is made of cubes.



New Task +: A figure is made of cubes.



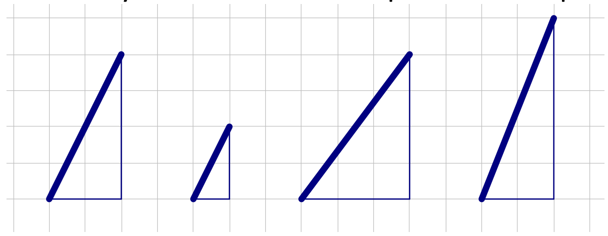
Notice that you may even remove the *directions*! Encourage students to come up with their own directions. For example, they may decide either to draw or to make the figure (or both). They may compare and contrast their solutions (Strategy 5), and explain what causes the similarities and differences (Strategy 3).

Grade 8

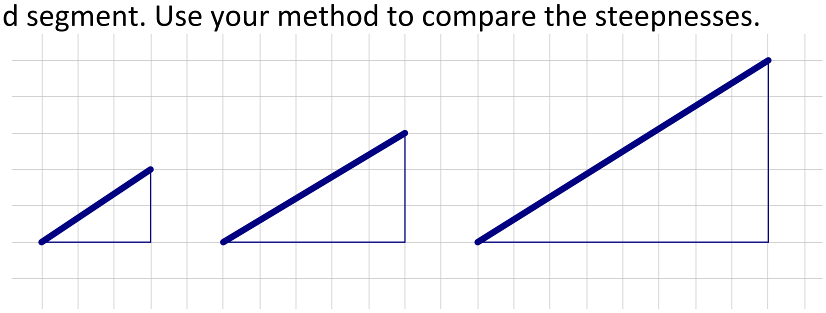
Strategy 8: Solve to learn.

Original task: Find the slope of the line. (It could be any line with a slope that is reasonably simple to describe.)

New Task: Invent a way to calculate a number that describes the steepness of each bold segment. Use your method to compare the steepnesses.



New Task +: Invent a way to use a number to describe the steepness of each bold segment. Use your method to compare the steepnesses.



This is the only example for Strategy 8 in which the new task is different than the original. Still, the basic idea is the same: students invent a method for describing slope *before* they are taught a formula. The light vertical and horizontal segments hint that they should try using these lengths in their calculations. They need to explore different ways of combining the two numbers so that steeper slopes always have larger numbers (and equal slopes have equal numbers)! The New Task + is more complex, because the slopes are hard to distinguish visually.

Strategy 9: Build a pattern.

Original task: Find the value of 3^{-2}.

New Task: Create a pattern to help you find the value of 3^{-2}.

New Task +: Create a pattern to help you find the value of $(0.5)^{-2}$.

The New Tasks may work best if combine them with Strategy 8: Solve to Learn. (Give the tasks to students before you teach them rules for evaluating negative exponents.) A helpful pattern to look at is

$3^4 = 81 \quad 3^3 = 27 \quad 3^2 = 9 \quad 3^1 = 3$, etc.

Grade 8

Strategy 10: Ask "What if…?"

Original task: Solve the equation.

$$5(x - 3) = 12$$

New Task: What happens to the solution if I increase 3 by 1? What if I keep increasing it by 1? What if I start with the 5 and keep increasing it by 1?

New Task +: Make similar types of changes to a more complex equation.

This New Task works very well with strategies 9 and 3, because students can look for patterns in the way that the solutions change and explain what causes them.