Deep Math Projects
Sharing Marbles

Some math problems have one answer, some have many answers, and some have no answer! Find as many answers as you can to these problems and show how you figured them out. If a problem does not have an answer, explain why.

Pablo has some marbles in a bag. He wants to share all of them equally with his friends.

1. Pablo has fewer than 30 marbles in his bag.

   He shares them between 3 friends and there is one marble left.
   He shares them between 4 friends, and there is one marble left.

   How many marbles were in the bag?

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2. Pablo has fewer than 50 marbles in his bag.

   He shares them between 3 friends and there is one marble left.
   He shares them between 4 friends, and there is one marble left.
   He shares them between 5 friends, and there are no marbles left.

   How many marbles were in the bag?

3. Pablo has fewer than 100 marbles in his bag.

   He shares them between 3 friends and there is one marble left.
   He shares them between 4 friends, and there is one marble left.
   He shares them between 5 friends, and there is one marble left.
   He shares them between 6 friends, and there are no marbles left.

   How many marbles were in the bag?

4. Pablo has many marbles in his bag.

   He shares them between 3 friends and there is one marble left.
   He shares them between 4 friends, and there is one marble left.
   He shares them between 5 friends, and there is one marble left.
   He shares them between 6 friends, and there is one marble left.
   He shares them between 7 friends, and there are no marbles left.

   How many marbles were in the bag?

5. Create your own problem about sharing marbles. Then solve it!

Sharing Marbles

Math Notes and Solutions

1. There may have been either 13 or 25 marbles in the bag. (Some students may include an answer of 1 as well, which is fine. This is an interesting discussion!)

   Possibilities for sharing them between 3 friends with one remaining:

   (1), 4, 7, 10, **13**, 16, 19, 22, **25**, 28

   Possibilities for sharing them between 4 friends with one remaining:

   (1), 5, 9, **13**, 17, 21, **25**, 29

   13 and 25 (and possibly 1) are the numbers common to both lists.

   Sample strategies for finding the numbers in the first list:

   Strategy 1: Make a list of multiples of 3 and add 1 to each.
   (0), 3, 6, 9, 12, 15, etc. becomes (1), 4, 7, 10, 13, 16, etc.

   Strategy 2: Multiply any whole number by 3, and then add 1. For example:
   (0 x 3 + 1 = 0 + 1 = 1)
   1 x 3 + 1 = 3 + 1 = 4
   2 x 3 + 1 = 6 + 1 = 7
   3 x 3 + 1 = 9 + 1 = 10, etc.

   Strategy 3: Use physical objects such as counters to create a whole number of groups of 3. Then add one more counter.

Possible Extension: Suppose that there were many (an unlimited number of) marbles? How many answers are there now? Do you see any patterns?

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2. There were 25 marbles in the bag. (Again, some students may include 1 as a solution.)

   This problem may go more quickly than the first one if students begin with their lists from the first problem. However, they should extend them, because there are more marbles than before.

   Three friends – one remaining:
   4, 7, 10, 13, 16, 19, 22, **25**, 28, 31, 34, 37, 40, 43, 46, 49
   Four friends – one remaining:
   5, 9, 13, 17, 21, **25**, 29, 33, 37, 41, 45, 49
   Five friends – none remaining:
   5, 10, 15, 20, **25**, 30, 35, 40, 45

   13 is no longer a solution, because it is not a multiple of 5. However, 25 works for 3, 4, or 5 friends. (Many students will not need to list the multiples of 5. They will simply notice that the ones digit must be 0 or 5.)

3. There is no solution.

   As before, students might begin by making lists for each number of friends.

   3 friends: 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, 43, 46, 49, 52, etc.
   4 friends: 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61, 65, 69, etc.
   5 friends: 6, 11, 16, 21, 26, 31, 36, 41, 46, 51, 56, 61, 66, 71, 76, 81, 86, etc.
   6 friends: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96

   They may eventually see that there is no number common to all four lists! The problem is with the first and last lists (for 3 friends and the 6 friends).

   To see why, try arranging the numbers in an array like this. Ask students to circle the numbers that are 1 greater than a multiple of 3. Then have them box the multiples of 6.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30

The numbers that have a remainder of 1 after dividing by 3 are always in the first and fourth columns. However, the multiples of 6 are all in the sixth column. (The basic problem is that every multiple of 6 is also a multiple of 3.)

4. The smallest solution is 301.

   Some students may find other solutions: 721, 1141, 1561, 1981, etc. Each answer increases by 420.

   This is a challenging problem with many strategies.

   Numbers that have no remainder when divided by 2, 3, 4, and 5 are

   60, 120, 180, 240, 300, 360, 420, etc.

   Numbers that have a remainder of one when you divide them by 2, 3, 4, and 5 are one greater than these:

   61, 121, 181, 241, 301, 361, 421, etc.

   The first one of these that is divisible by 7 is 301.