Deep Algebra Projects: Pre-Algebra/Algebra
Factory Functions

There are five machines in a factory, which are located along a 50-yard aisle at distances of 8, 14, 18, 31 and 41 yards from one wall.

Stage 1

Solve Problem #1 and #2 by completing steps 1 – 11 in the checklist.

1. A new machine is to be located somewhere along this aisle. Items produced by the existing machines are brought to the new machine for further processing. The same number of items will be brought to the new machine from each of the five existing machines at a cost of $1 per yard for each item moved. Your job is to locate the new machine so that the total cost of this operation will be the least possible.

2. Suppose the machine originally located at 18 yards from the wall is removed. Redo the problem under these conditions.

Stage 2

3. Complete steps 12 and 13 in the checklist for problems 1 and 2. For step 12, be sure to explain your thinking process for finding the formulas.

Stage 3

4. Show how to find formulas for the *minimum* cost for five machine and four machine scenarios. Test your formulas by trying machines in different locations.

5. Show how to create a formula for the minimum cost based on any number of machines. Explain your thinking.

inspired by "Fantastik 21: A Locatshun Problem" in *Fantastiks of Mathematics*, by Cliff Sloyer. Janson Publications, 1986

Best Location for a Machine – Checklist

Stage 1

1. ___________ Show sample calculations of cost for 2 separate locations.

2. ___________ Make a Location/Cost Table for 0 – 50 yards.

3. ___________ Describe any patterns in your table.

4. ___________ Explain why the patterns happen.

5. ___________ Make a Location (horizontal axis)/Cost (vertical axis) *line* graph for 0 – 50 yards. Use graph paper. Title the graph, and show the scales and labels on the axes.

6. ___________ Remove the machine located at 18 yards and repeat steps 1 – 5. (Step 1)

7. ___________ (Step 2)

8. ___________ (Step 3)

9. ___________ (Step 4)

10. ___________ (Step 5)

11. ___________ Describe a shortcut for finding the best location for the new machine given any number of existing machines.

Stage 2

12. ___________ Go back to the original problem with machines at 8, 14, 18, 31 and 41 yards. Label the new machine's location as the variable *x.* Write formulas for the cost at different locations. (You will need a new formula every time the pattern in your original table changes.) Simplify each formula as much as possible by combining like terms.

13. ___________ Compare your formulas to your original table and graph. Describe any connections that you see.

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Best Location for a Machine
Solutions

1. <u>5 machines</u> (at 8, 14, 18, 31, and 41 yards from the wall):

The best location is at 18 yards, (or as close as you can get, since there is already a machine there).

Sample Calculations:

10 yards (10 – 8) + (14 – 10) + (18 – 10) + (31 – 10) + (41 – 10) =
2 + 4 + 8 + 21 + 31 = 66

25 yards (25 – 8) + (25 – 14) + (25 – 18) + (31 – 25) + (41 – 25) =
17 + 11 + 7 + 6 + 16 = 57

45 yards (45 – 8) + (45 – 14) + (45 – 18) + (45 – 31) + (45 – 41) =
37 + 31 + 27 + 14 + 4 = 113

<u>Cost for 5 Machines</u>

Loc (yds)	Cost ($)
0	112
1	107
2	102
3	97
4	92
5	87
6	82
7	77
8	72
9	69
10	66

Loc (yds)	Cost ($)
11	63
12	60
13	57
14	54
15	53
16	52
17	51
18	50
19	51
20	52

Loc (yds)	Cost ($)
21	53
22	54
23	55
24	56
25	57
26	58
27	59
28	60
29	61
30	62

Loc (yds)	Cost ($)
31	63
32	66
33	69
34	72
35	75
36	78
37	81
38	84
39	87
40	90

Loc (yds)	Cost ($)
41	93
42	98
43	103
44	108
45	113
46	118
47	123
48	128
49	133
50	138

The cost *decreases* at a constant rate of

- $5 per yard between 0 and 8 yards.
- $3 per yard between 8 and 14 yards.
- $1 per yard between 14 and 18 yards.

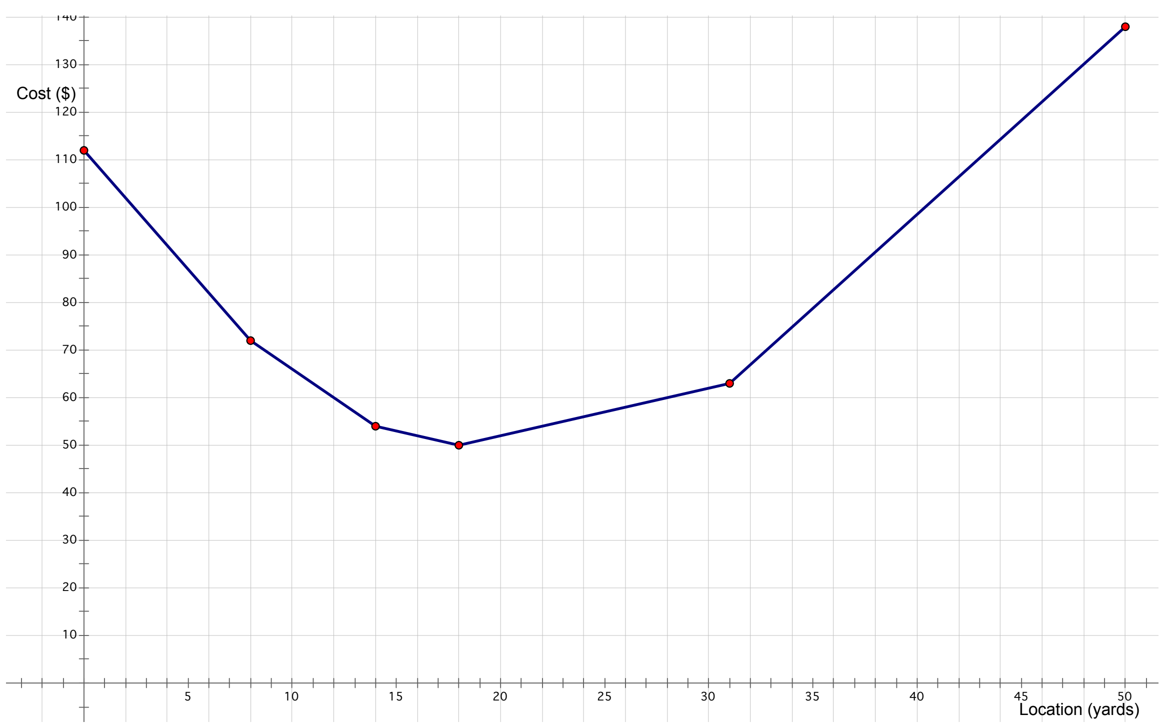
The cost *increases* at a constant rate of

- $1 per yard between 18 and 31 yards.
- $3 per yard between 31 and 41 yards.
- $5 per yard between 41 and 50 yards.

inspired by "Fantastik 21: A Locatshun Problem" in *Fantastiks of Mathematics*, by Cliff Sloyer. Janson Publications, 1986

When you imagine moving the new machine one yard to the right, it gets farther away from the machines to the left of it and closer to the machines to the right of it. This causes patterns in the total distance between the machines and therefore in the cost. For example, if you move the machine from the 16- to the 17-yard position, it gets 1 yard farther from the two machines and 8 and 14 yards, which increases the cost by $2. However, you get closer to the three machines at 18, 31, and 41 yards, which decreases the cost by $3. The net effect is a $1 decrease in cost. This occurs everywhere 14 and 18 yards, where there are two machines to the left and three machines to the right.

Location vs. Cost for 5 Machines



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inspired by "Fantastik 21: A Locatshun Problem" in *Fantastiks of Mathematics*, by Cliff Sloyer. Janson Publications, 1986

2. <u>4 Machines</u> (located at 8, 18, 31, and 41 yards from the wall):

The best locations are anywhere between 14 and 31 yards, where the cost is $50!

Sample Calculations:

10 yards: (10 – 8) + (14 – 10) + (31 – 10) + (41 – 10) = 2 + 4 + 21 + 31 = 58

25 yards: (25 – 8) + (25 – 14) + (31 – 25) + (41 – 25) = 17 + 11 + 6 + 16 = 50

45 yards: (45 – 8) + (45 – 14) + (45 – 31) + (45 – 41) = 37 + 31 + 14 + 4 = 86

<u>Cost for 4 Machines</u>

Loc (yds)	Cost ($)	Loc (yds)	Cost ($)	Loc (yds)	Cost ($)	Loc (yds)	Cost ($)	Loc (yds)	Cost ($)
0	94								
1	90	11	56	21	50	31	50	41	70
2	86	12	54	22	50	32	52	42	74
3	82	13	52	23	50	33	54	43	78
4	78	14	50	24	50	34	56	44	82
5	74	15	50	25	50	35	58	45	86
6	70	16	50	26	50	36	60	46	90
7	66	17	50	27	50	37	62	47	94
8	62	18	50	28	50	38	64	48	98
9	60	19	50	29	50	39	66	49	102
10	58	20	50	30	50	40	68	50	106

The cost *decreases* at a constant rate of

- $4 per yard between 0 and 8 yards.
- $2 per yard between 8 and 14 yards.

The cost remains *constant*

- between 14 and 31 yards.

The cost *increases* at a constant rate of

- $2 per yard between 31 and 41 yards.
- $4 per yard between 41 and 50 yards.

Again, when you imagine moving the new machine one yard to the right, it gets farther away from the machines to the left of it and closer to the machines to the right of it. For example, if you move the machine from the 16- to the 17-yard position, it gets 1 yard farther from the two machines and 8 and 14 yards, which increases the cost by $2. However, you get closer to the two machines at 31 and 41 yards, which decreases the cost by $2. The net effect is no change in cost. This occurs everywhere 14 and 31 yards, because there are two machines to the left and two machines to the right.

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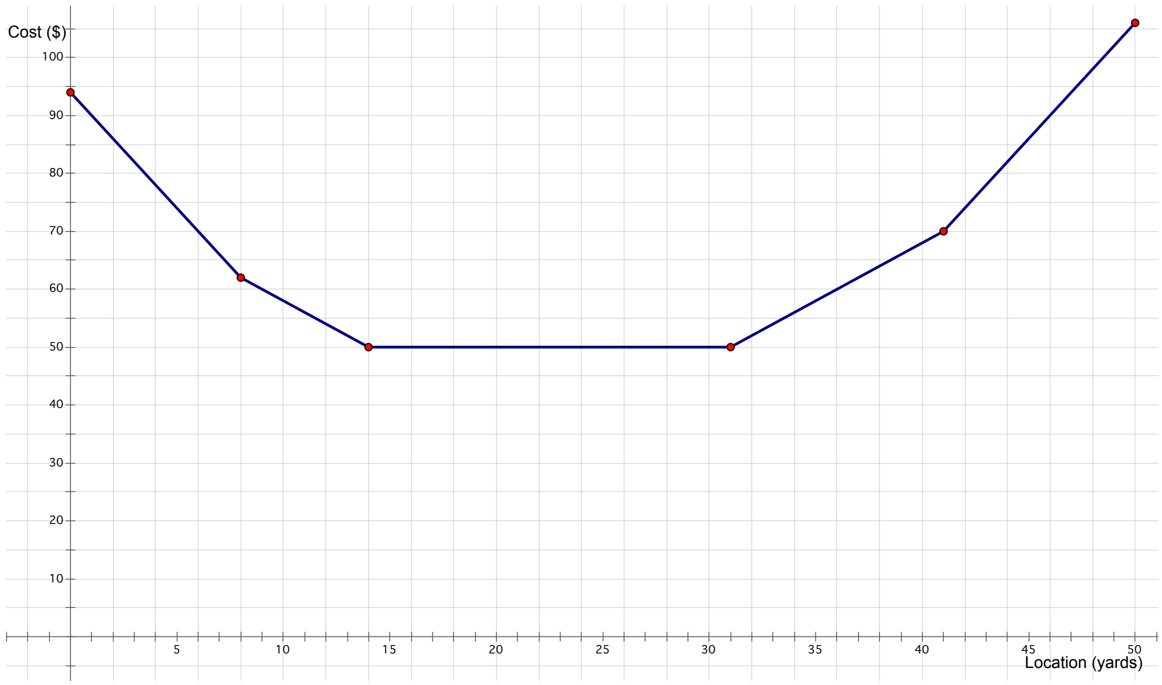
Location vs. Cost for 4 Machines



The best location for the new machine is the *median* location of the existing machines (or anywhere between the two middle machines if there is an even number of machines).

3. Formulas:

between 0 and 8 yards: C = 112 – 5 • x
from (8 – x) + (14 – x) +(18 – x) +(31 – x) +(41 – x)

between 8 and 14 yards: C = 96 – 3 • x
from (x – 8) + (14 – x) +(18 – x) +(31 – x) +(41 – x)

between 14 and 18 yards: C = 68 – x
from (x – 8) + (x – 14) +(18 – x) +(31 – x) +(41 – x)

between 18 and 31 yards: C = 32 + x
from (x – 8) + (x – 14) +(x – 18) +(31 – x) +(41 – x)

between 31 and 41 yards: C = –30 + 3 • x
from (x – 8) + (x – 14) +(x – 18) +(x – 31) +(41 – x)

between 41 and 50 yards: C = –112 + 5 • x
from (x – 8) + (x – 14) +(x – 18) +(x – 31) +(x – 41)

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Students' formulas may differ from these and still be correct. The expressions for the cost must simply be algebraically equivalent to the given expressions.

Students will also have many strategies for finding these formulas. Some may write the long expressions shown above and simplify them as suggested. Others may experiment with the patterns in the tables until they find something that works. Occasionally, they may use more sophisticated reasoning by paying attention to the amount by which the constant terms in the formulas change when you pass a machine. Students who are familiar with "starting numbers" (*y*-intercepts) may extend the line segments in the graph to see where they cross the *y*-axis.

Students who have studied rates of change should notice that these numbers appear in the formulas as the coefficients of x (the numbers multiplied by x).

4. If five machines are a, b, c, d, and e yards from wall (from left to right), then the minimum cost, C_{min}, is

$$C_{min} = (e - a) + (d - b)$$
or
$$C_{min} = (e + d) - (a + b)$$

For six machines at distances of a, b, c, d, e, and f, the minimum cost is

$$C_{min} = (f - a) + (e - b) + (d - c)$$
or
$$C_{min} = f + e + d - (a + b + c)$$

Many students are likely to discover the formulas by experimenting with the values of the cost and the locations of the machines and looking for patterns. Some may provide written explanations of why the formulas make sense. A few may be able to use algebra to prove their answers.

5. Subtract the location of the leftmost machine from that of the rightmost machine. Move 1 machine toward the center on both sides and find this (positive) difference as well. Continue the process until you reach the middle. Add each of these differences. If there is an odd number of machines, ignore the machine in the middle. (You are essentially subtracting its location from itself.)

Alternatively, you may subtract the sum for all machines to the left of the median location from the sum for all machines to right of it.

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